PERFORMANCE EVALUATION EXERCISES

TESTS

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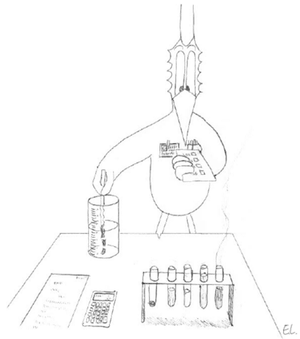

1. If the data is in the critical region we...
   (a) ☐ Accept H_0
   (b) ☐ Reject H_0
   (c) ☐ It depends on the nature of the test
   (d) ☐ It depends on the size of the test

2. Saying that a test is of size 5% means that...
   (a) ☐ The probability to accept H_0 when H_0 does not hold is ≤ 0.05
   (b) ☐ The probability to reject H_0 when H_0 it holds is ≤ 0.05
   (c) ☐ Both

3. If the $p-$value of a test is small we ...
   (a) ☐ Accept H_0
   (b) ☐ Reject H_0
   (c) ☐ It depends on the nature of the test
   (d) ☐ It depends on the size of the test

4. We have a collection of random variables X_i, Y_i which correspond to non paired simulation results with configuration 1 or 2. How can you test whether the configuration plays a role or not ?
   (a) ☐ With a Wilcoxon Rank Sum test
   (b) ☐ With an ANOVA test
   (c) ☐ With either
   (d) ☐ With none

5. We test whether a distribution is gaussian using a Kolmogorov-Smirnov test against the fitted distribution. We obtain a $p-$value.
   (a) ☐ The true $p-$value is smaller
   (b) ☐ We have obtained the true $p-$value
   (c) ☐ The true $p-$value is larger

(d) ☐ It depends on the data

6. We have two data sets X_i and Y_j believed to be iid and from one exponential distribution each. We want to test whether the parameter of their exponential distribution is the same.

   Give the design of a corresponding likelihood ratio test. Give a formula for the $p-$value when m, n are large.

7. We have some data set $\vec{Y} = Y_{i=1:I}$ modelled with a parametric model with $\theta \in \Theta$. Let $f_{\vec{Y}}(\vec{y}|\theta)$ be the PDF of the observation $\vec{y} = y_{1:I}$. We assume that we have a method to compute $\hat{\theta}(\vec{y})$, the maximum likelihood estimator of θ for value of the data set $\vec{y}$.

   (a) Give a likelihood ratio test for the test

   $H_0 : \theta = \theta_0$ versus $H_1 : \theta \in \Theta$

   (b) Give the pseudo-code of an algorithm to compute the $p-$value of this test using Monte-Carlo simulation with R runs.

   (c) We run this algorithm with $R = 10'000$ and find $p = 0$. Give a 99% confidence for the true $p-$value. What can we conclude at a size of 5% ?

8. We consider again the case in the previous question. Using Monte-Carlo simulation, we have obtained a 99% confidence interval $[\ell(\vec{y}), u(\vec{y})]$ for the $p-$value. We reject H_0 if the true p is small, but since we don't know the true $p-$value, we use the rejection condition $u(\vec{y}) < \alpha$. What value of α should we chose to ensure that this way of doing provides a test of size 5% ?