PERFORMANCE EVALUATION EXERCISES

PALM CALCULUS

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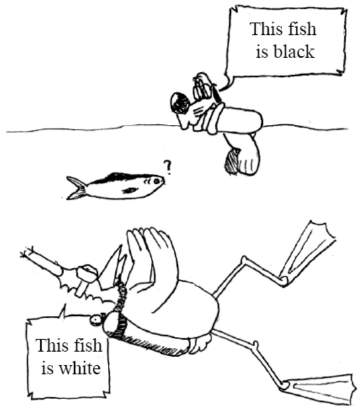


1. SovRail claims that only 5% of trains arrivals are late. BorduKonsum claims that 30% of train users suffer from late train arrivals. Say which is true (Hint: Use the heuristic seen in class; you may indroduce the variables $D_n = 1$ if the nth arrival is late and $D_n = 0$ if it is on time):

   (a) ☐ At least one of them uses alternative facts.

   (b) ☐ Number of passengers in a late train $\approx 1.15\times$ number of passengers in a train

   (c) ☐ Number of passengers in a late train $\approx 2.45\times$ number of passengers in a train

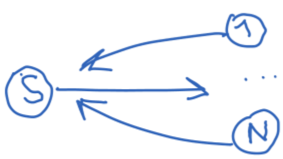
   (d) ☐ Number of passengers in a late train $\approx 6\times$ number of passengers in a train

2. Instagram uses a clustering algorithm to classify the vacation preferences of their N users and obtains M clusters. (We have $N > M \gg 1$). We have obtained the distribution of the number of users per cluster and found that it follows an exponential distribution with rate λ. What is the PDF of the size of the cluster seen by an arbitrary user ? Compare the mean size of a cluster seen by a user and the mean size of an arbitrary cluster.

   Recall that the exponential distribution with rate λ has mean $\frac{1}{\lambda}$, variance $\frac{1}{\lambda^2}$ and PDF $f_C(x) = \lambda e^{-\lambda x}\mathbf{1}_{\{x\geq 0\}}$.

3. A sensor sends a broadcast poll and waits for all clients to ACK.

   The round trip times $S \to i \to S$ are iid Exp(1) and N is large. How many polls per time unit are sent ?



   (a) ☐ $\lambda \approx \frac{1}{\log N}$

   (b) ☐ $\lambda \approx \frac{1}{\sqrt{N}}$

   (c) ☐ $\lambda \approx \frac{1}{N}$

   (d) ☐ $\lambda \approx \frac{1}{N^2}$

4. Same question if the round trip times are iid standard Pareto with index $p = 2$.

   (a) ☐ $\lambda \approx \frac{1}{\log N}$

   (b) ☐ $\lambda \approx \frac{1}{\sqrt{N}}$

   (c) ☐ $\lambda \approx \frac{1}{N}$

   (d) ☐ $\lambda \approx \frac{1}{N^2}$

5. For the random waypoint model, the distribution of the next waypoint is uniform...

   (a) ☐ when sampled at an arbitrary waypoint

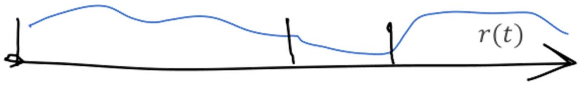
   (b) ☐ when sampled at an arbitrary point in time

   (c) ☐ both

   (d) ☐ none

6. A system downloads data over a channel with a fluctuating instantaneous rate $r(t)$. The data is sent in rounds, of average duration $\bar{T}$. The average amount of data transferred in one round is $\bar{B}$.

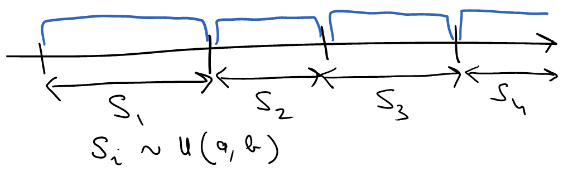
   We sample the channel using a Poisson process of rate λ and find a sampled data transfer rate $\bar{r}$.



   Give a formula for the value of $\bar{r}$ as a function of $\bar{T}$, $\bar{B}$ and λ. (Hint: use the large-time heuristic or the inversion formula.)

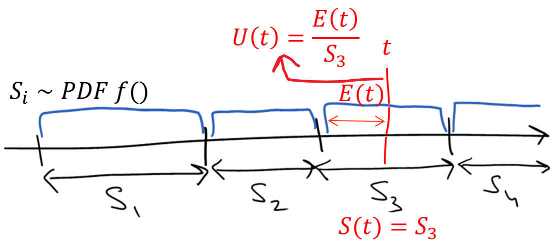
7. Sensing devices send one message at the end of the sensing interval.

   The durations of sensing intervals are iid uniform between a and b. We want to write a perfect simulation of a sensor. From which distribution should we sample the residual cycle time ?



8. (Continuation) We continue with the previous example but now assume that the sensing intervals are iid with common pdf f (not necessarily uniform).

   We call $S(t)$ and $E(t)$ the durations of the current interval and of the time elapsed since the beginning of the current interval, when observed at time t. Also let $U(t) = \frac{E(t)}{U(t)} \in [0; 1]$ be the fraction of the elapsed interval observed at time t.



   (a) Under which condition does this simulation have a stationary regime ?

   (b) When this condition holds, what is the distribution of $U(t)$?

(c) Are $U(t)$ and $S(t)$ independent, i.e. is the fraction of the interval we are in independent of the length of the interval ?

9. (Continuation) An algorithm for the perfect simulation of a device is returning the duration of the current interval $S(t)$ and the residual time $R(t)$ until end of current interval. Which implementation is correct ?

A.
1. Draw S from distribution with PDF $\lambda s f(s)$
2. Draw $V \sim$ Unif$(0, 1)$
3. $R = VS$

B.
1. Draw S from distribution with PDF $f(s)$
2. Draw $V \sim$ Unif$(0, 1)$
3. $R = (1 - V)S$

(a) ☐ A
(b) ☐ B
(c) ☐ Both
(d) ☐ None

10. Consider the random waypoint model, where the speed chosen at a waypoint is sampled from the pdf $f()$. Can we choose $f()$ such that

(a) the model has a stationary regime
(b) the distribution of speed sampled at an arbitrary point in time is uniform between 0 and $v_{\max}$?