EPFL

# An Introduction to Network Calculus

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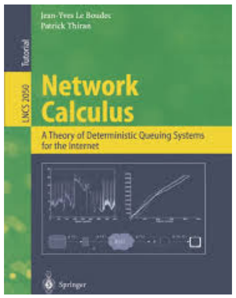
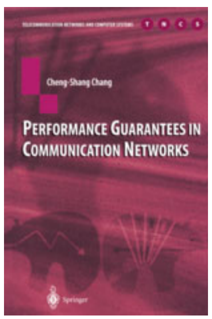
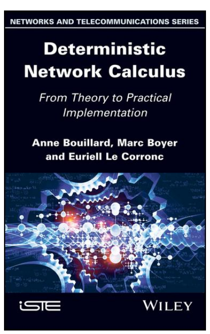
2019 November 12

# Contents

# What is Network Calculus ?




A theory and tools to compute bounds on queuing delays, buffers, burstiness of flows, etc.

R Cruz, CS Chang, JY Le Boudec, P Thiran, …

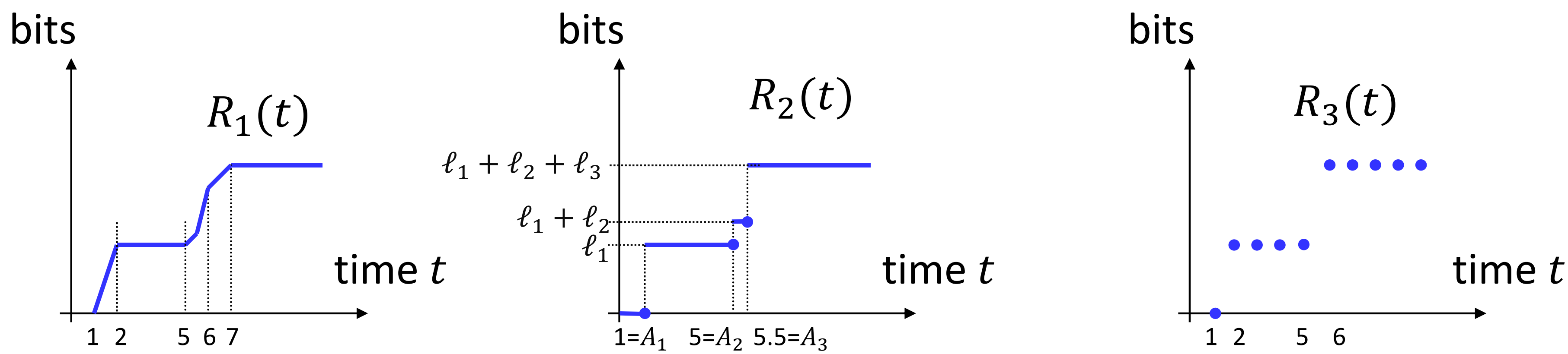
For deterministic networking, per-flow and per-class queuing

Derive system equations ⇒ formal proofs

Stochastic extensions exist (not discussed here)

# 1. Representation of Data Flow

Cumulative flow: $R(t)$, non-decreasing with $R(0) = 0$

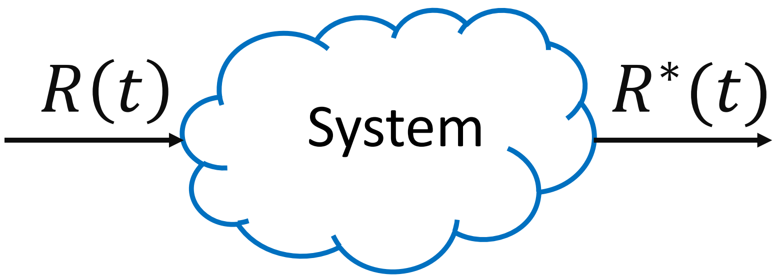


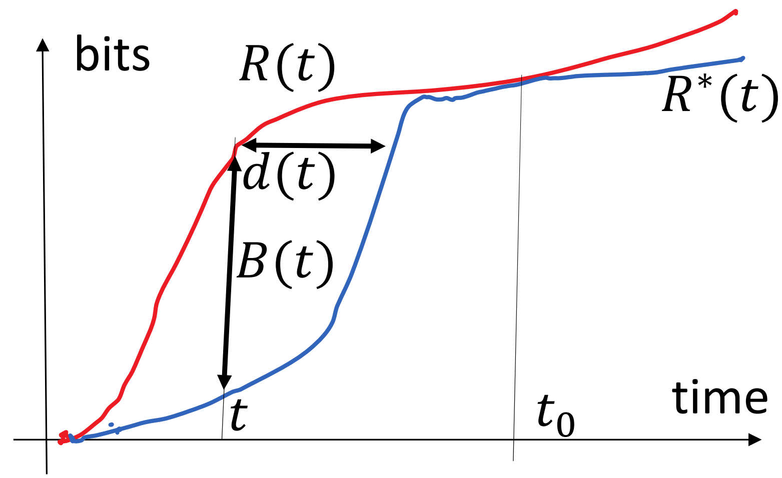

Fluid model (continuous) | Packet train (left continuous) | Discrete-time

Daters: $(A, L)$ with $A = (A_1, A_2, \dots)$ (dates) and $\ell = (\ell_1, \ell_2, \dots)$ (lengths in bits)

For a packet train: $R(t) = \sum_{n\geq 1} \ell_n 1_{\{A_n < t\}}$

# Delay and Backlog




Backlog at time $t$ = $R(t) - R^*(t)$

If System preserves order for this flow: Delay $\leq h(R, R^*)$

with $h(R, R^*) = \sup_t d(t)$

and $d(t) = \inf\{d \text{ s.t.} R(t) \leq R^*(t + d)\}$

(horizontal deviation)

# 2. Arrival Curve

Flow with cumulative function $R(t)$ has $\alpha$ as (maximal) arrival curve if

$$R(t) - R(s) \leq \alpha(t - s) \text{ for any } t \geq s \geq 0$$

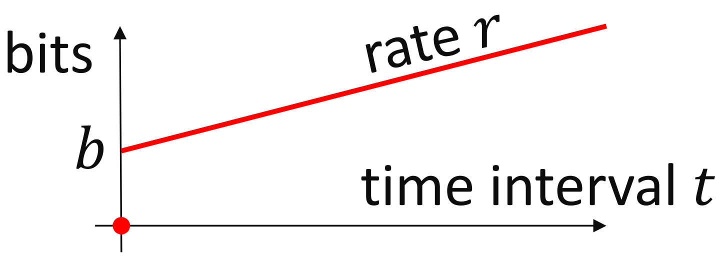
where $\alpha$ is a monotonic nondecreasing function $\mathbb{R}^+ \to [0, +\infty]$

token bucket constraint $(r, b)$

(leaky buket constraint)

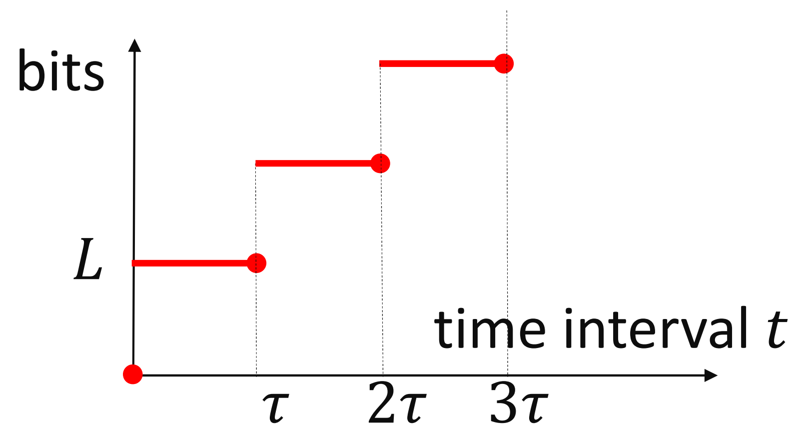
with rate $r$ and burst $b$:

$\alpha(t) = rt + b$




[R. Cruz, PhD Dissertation 1987]

periodic stream of packets of size $\leq L$: $\alpha(t) = L \left\lceil \frac{t}{\tau} \right\rceil$



# Aggregation Property

If every flow $f$ has arrival curve $\alpha_f$ then the aggregation $R = \sum_f R_f$ has arrival curve $\sum_f \alpha_f$

If every flow $f$ is token-bucket constrained $(r_f, b_f)$ then the aggregation is token-bucket constrained $(\sum_f r_f\ ,\ \sum_f b_f)$

# Min-Plus Convolution of wide-sense increasing functions $[0;+\infty) \to [0;+\infty]$

$$f(t) = \inf_{s \geq 0} \left( f_1(s) + f_2(t-s) \right)$$

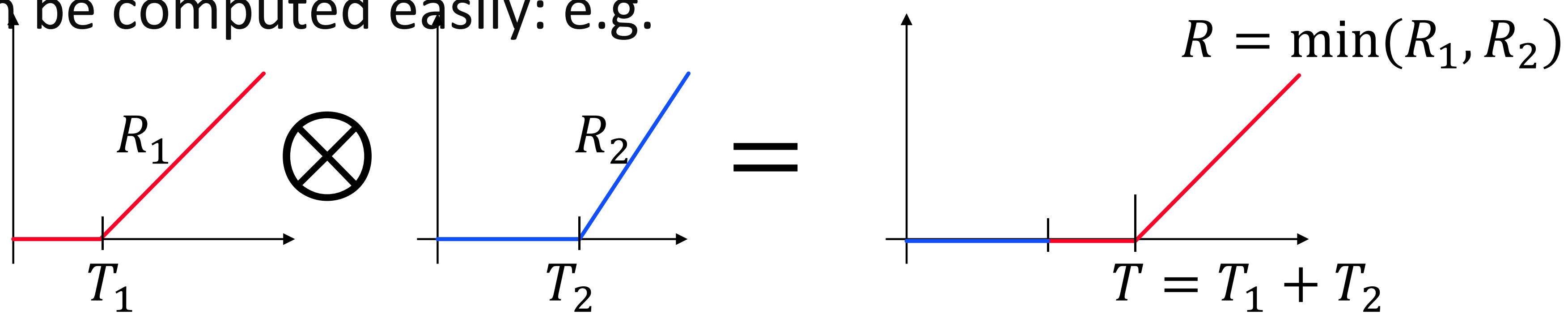
$$f = f_1 \otimes f_2$$

This operation is called *min-plus convolution*. It has the same nice properties as usual convolution; e.g.

$$(f_1 \otimes f_2) \otimes f_3 = f_1 \otimes (f_2 \otimes f_3)$$

$$f_1 \otimes f_2 = f_2 \otimes f_1$$

It can be computed easily: e.g.



# Min-Plus Calculus

$\otimes$ is associative, commutative

Neutral element: $f \otimes \delta_0 = f$ where $\delta_0(0) = 0, \delta_0(t) = +\infty, t > 0$

$\otimes$ distributes w.r.t. min: $f \otimes (g \wedge h) = (f \otimes g) \wedge (f \otimes h)$

$\otimes$ is isotone: $f \leq f'$ and $g \leq g' \Rightarrow f \otimes g \leq f' \otimes g'$

Functions passing through the origin ($f(0) = g(0) = 0$):
$f \otimes g \leq f \wedge g$

Concave functions passing through the origin: $f \otimes g = f \wedge g$

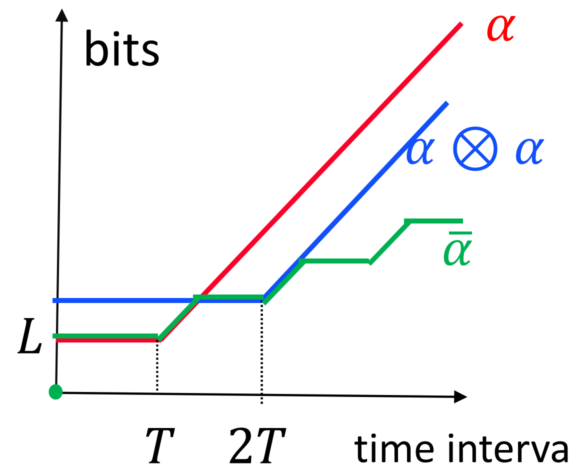
Convex piecewise linear functions: $f \otimes g$ is the convex piecewise linear function obtained by putting end-to-end all linear pieces of $f$ and $g$, sorted by increasing slopes

[Bouillard et al 2018, Chapters 3 and 4]

# Min-Plus Convolution and Arrival Curves

$\alpha$ is an arrival curve for $R \Leftrightarrow R(t) \leq R(s) + \alpha(t-s), \forall s \in [0,t]$
$$\Leftrightarrow R \leq R \otimes \alpha$$

Any arrival curve $\alpha$ can be replaced by its sub-additive closure
$\bar{\alpha} = \inf\{\delta_0, \alpha,\ \alpha \otimes \alpha,\ \alpha \otimes \alpha \otimes \alpha, \dots\}$
with $\delta_0(0) = 0,\ \ \delta_0(t) = +\infty$ for $t > 0$




$\bar{\alpha}$ is sub-additive, i.e. $\bar{\alpha}(s+t) \leq \bar{\alpha}(s) + \bar{\alpha}(t)$
and $\bar{\alpha}(0) = 0$

when $\alpha(0) = 0$,
$\alpha$ is an arrival curve for $R \Leftrightarrow \boxed{R = R \otimes \alpha}$

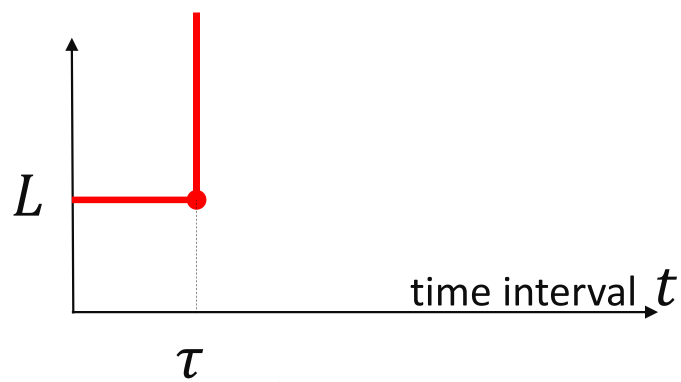
# Example of Sub-Additive Closure

Flow has at most $L$ bits in any interval of duration $\tau$

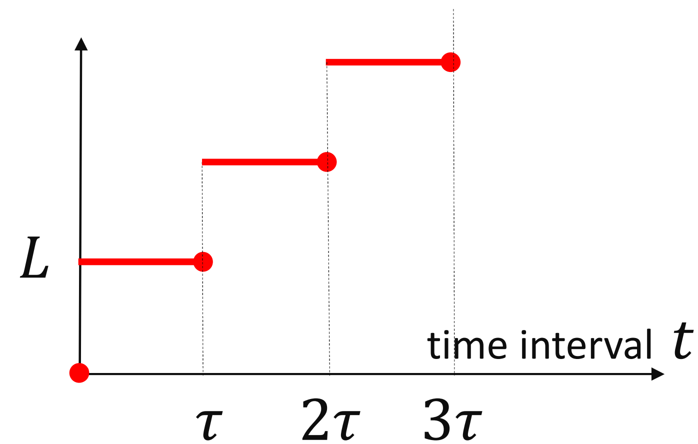
$\Leftrightarrow R(t+\tau) - R(t) \leq L$ for all $t$

$\Leftrightarrow$ flow has arrival curve $\alpha$

$\Leftrightarrow$ flow has arrival curve $\bar{\alpha}$

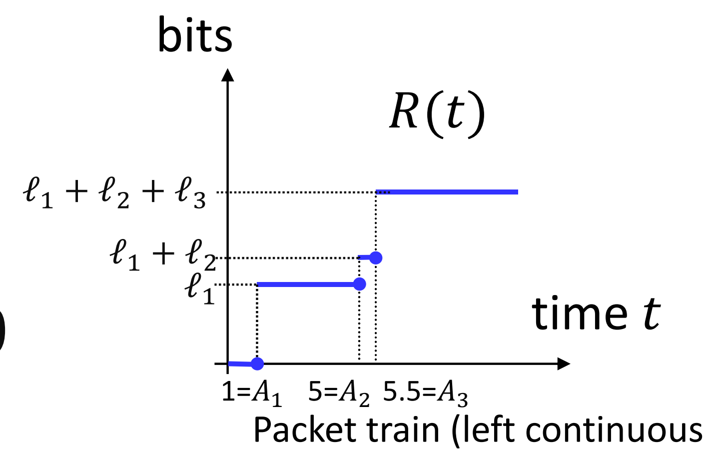



$$\alpha(t) = \begin{cases} L, t \leq \tau \\ +\infty, t > \tau \end{cases}$$




$$\bar{\alpha}(t) = L \left\lceil \frac{t}{\tau} \right\rceil$$

# Dater-Based (Max-Plus) Representation




For a left-continuous packet train, the arrival curve constraint

$R(t) - R(s) \leq \alpha(t - s)$ for any $t \geq s \geq 0$

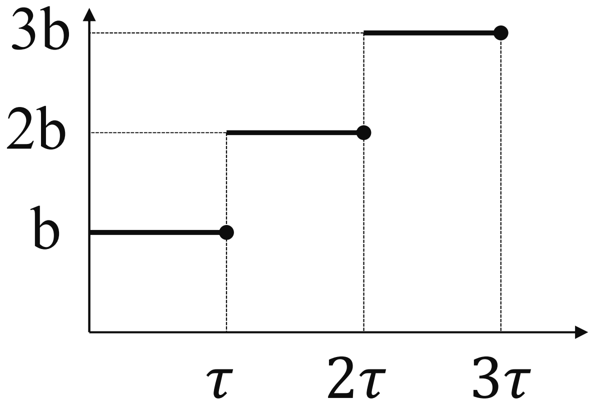
is equivalent to

$A_n - A_m \geq \alpha^{\downarrow}\left(\sum_{j=m}^{n} \ell_j\right)$ for all $1 \leq m \leq n$

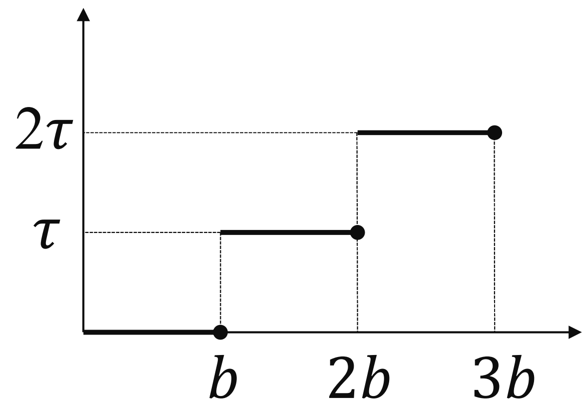
where $\alpha^{\downarrow}(x) = \inf\,\{t, \alpha(t) \geq x\}$ (lower pseudo-inverse) [Le Boudec 2018]

# Lower Pseudo-Inverse [Liebeherr 2017]

$$\alpha^{\downarrow}(x) = \inf\{t, \alpha(t) \geq x\}$$



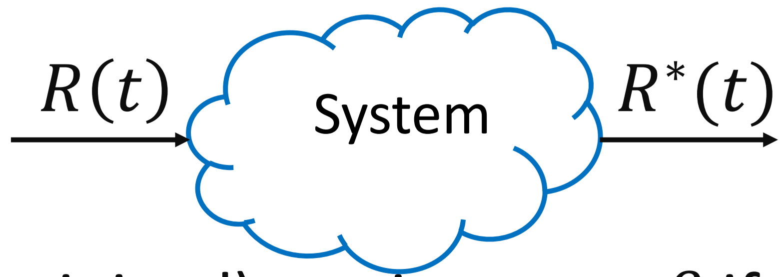

$$\alpha(t) = b\left\lceil \frac{t}{\tau} \right\rceil$$




$$\alpha^{\downarrow}(x) = \tau\left\lceil \frac{x}{b} - 1 \right\rceil$$

for $x > 0$

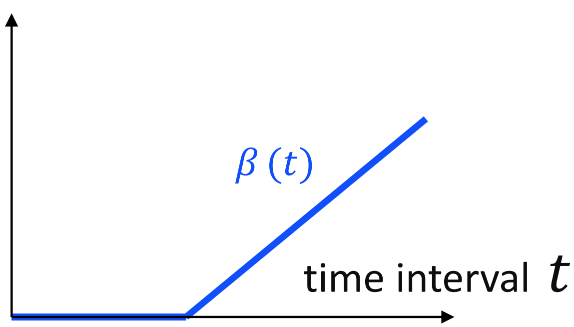
# 3. Service Curve

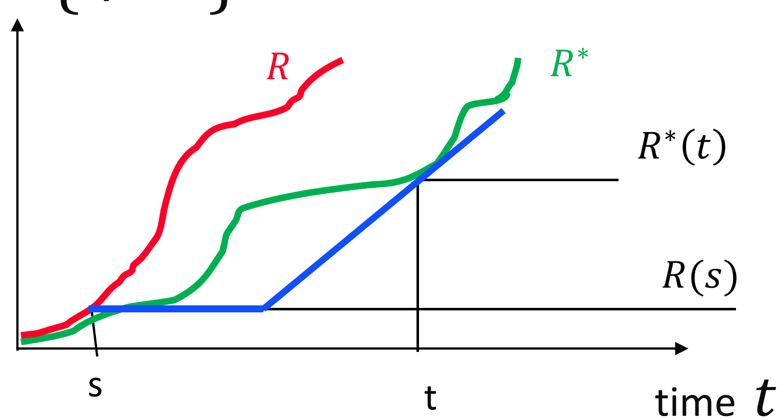



System offers to this flow a (minimal) service curve $\beta$ if $R^* \geq R \otimes \beta$, i.e. :

$$\forall t \geq 0, \exists s \in [0, t]: R^*(t) \geq R(s) + \beta(t - s)$$

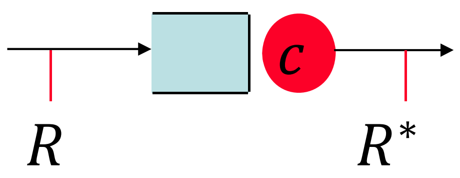
where $\beta$ is a function : $\mathbb{R}^+ \rightarrow \mathbb{R} \cup \{+\infty\}$

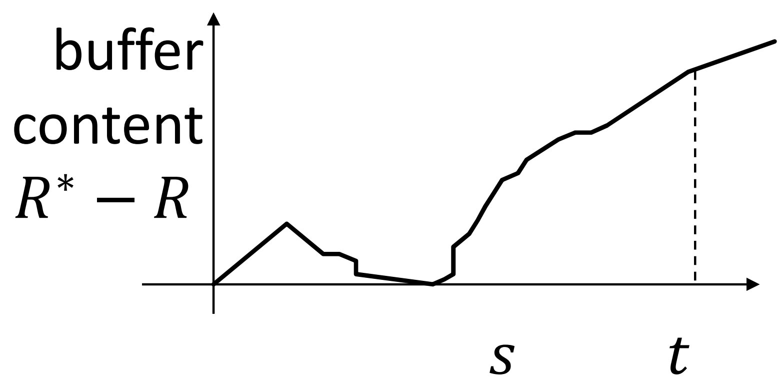


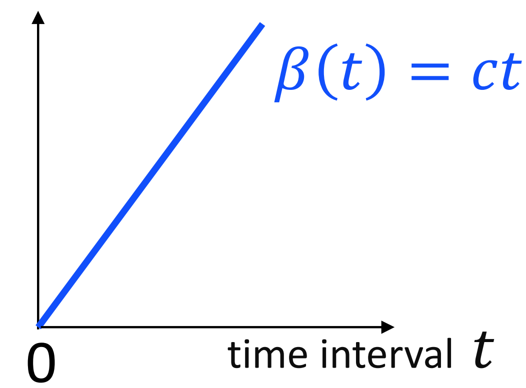




[Le Boudec 96, Chang 97, Bouillard et al 2018]

# The constant rate server offers service curve $\beta(t) = ct$



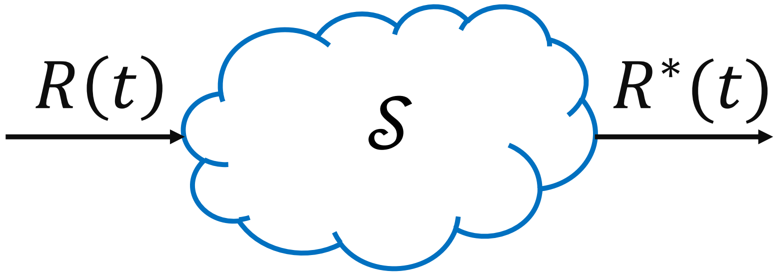

Proof: take $s$ = beginning of busy period:

$$R^*(t) - R^*(s) = c\,(t-s) \text{ and } R^*(s) = R(s)$$
$$\Rightarrow R^*(t) - R(s) = c\,(t-s)$$

$\beta(t) = ct$ is a service curve; it is also a *strict* service curve

[Cruz 95]

# Strict Service Curve

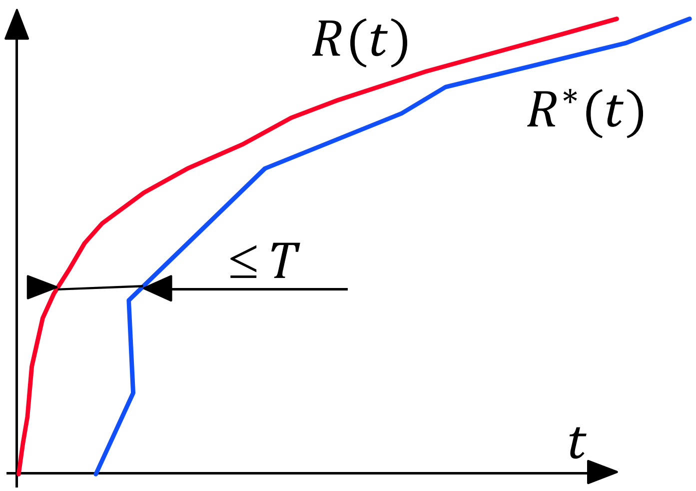


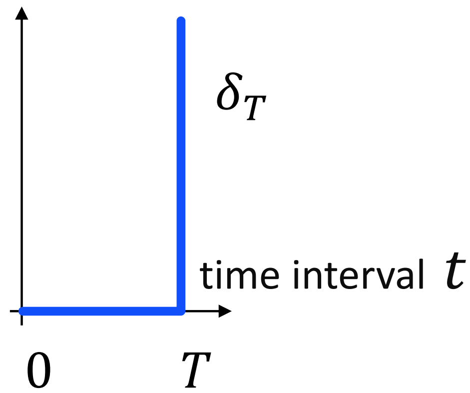

System $\mathcal{S}$ offers to a flow a strict service curve $\beta$ if for any $s < t$ inside a backlogged period, i.e. such that $R^*(u) < R(u), \forall u \in (s, t]$, we have $R^*(t) - R^*(s) \geq \beta(t - s)$

$\mathcal{S}$ is typically a single queuing point

strict service curve ⇒ service curve

# The guaranteed-delay node offers service curve $\delta_T$

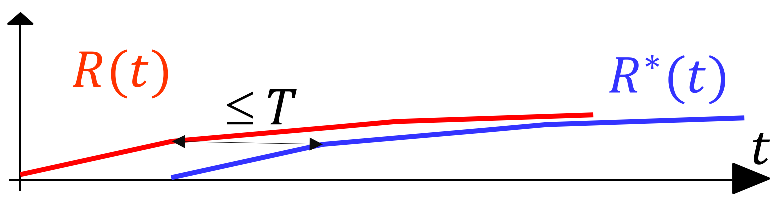



$$\delta_T(t) = 0 \text{ if } t \leq T$$
$$\delta_T(t) = +\infty \text{ if } t > T$$

For a node that is FIFO for this flow:
delay $\leq T$ $\Leftrightarrow$ nodes offers to this flow a service curve $\delta_T$

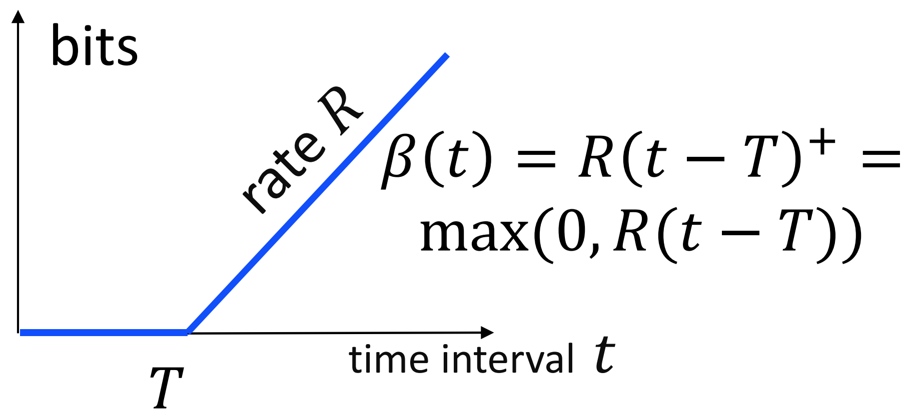
Not a strict service curve



# Service Curve Examples

Rate-latency service curve :

$$\beta(t) = R(t-T)^+$$




Example: Static Priority without pre-emption, fixed line rate $C$

High prio: $\beta_H(t) = (Ct - MTU_L)^+$

(strict service curve) ($MTU_L$ = max packet size, low prio)

Low prio: when high priority constrained by $\alpha(t) = rt + b, r < C$:

$\beta_L(t) = ((C-r)t - b)^+$ (not a strict service curve)

$\beta'_L(t) = \left((C-r)t - b - MTU_L\right)^+$ (strict service curve)

[Bouillard et al 2018]

# Service Curve Example: Deficit Round Robin

Popular per-flow scheduler, one queue per flow [Shreedhar and Varghese, 1995]

Visit every queue in sequence and at every visit, serve up to $Q_i$ bits.

DRR offers to flow $i$ a strict service curve:

$$\beta_i(t) = R_i(t - T_i)^+$$

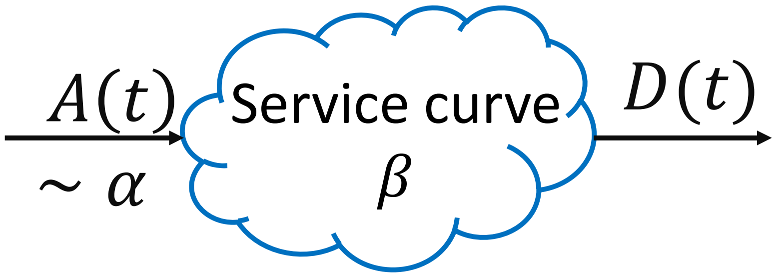
with $R_i = \frac{Q_i}{\sum_j Q_j} c, \quad T_i = \frac{\bar{Q}_i + \bar{L}_i}{c} + L_{\max,i}\left(\frac{1}{R_i} - \frac{1}{c}\right),$

$\bar{Q}_i = \sum_{j \neq i} Q_j, \;\; \bar{L}_i = \sum_{j \neq i} L_{\max,j}$ and $c$ is the line rate.

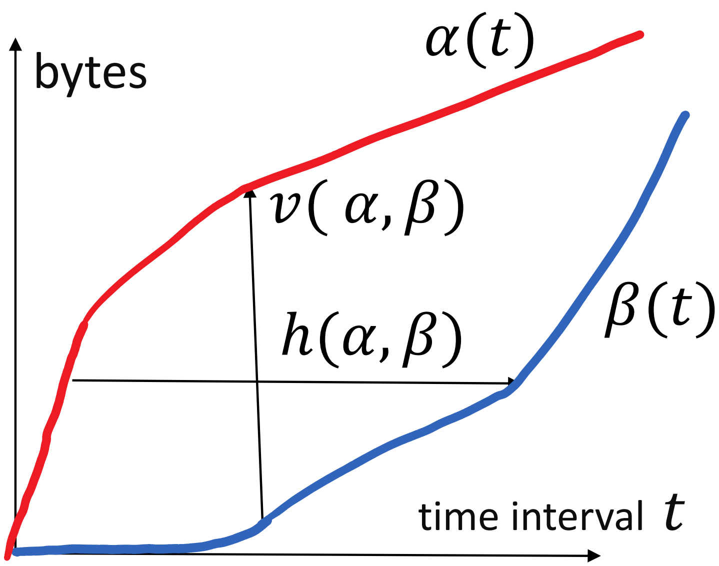
[Boyer et al 2012]

- Other examples: Packetized Generalized Processor Sharing, RFC 2212, IEEE AVB, IEEE TSN, etc. [De Azua – Boyer 2014] [Bouillard et al 2018]

# Three Tight Bounds

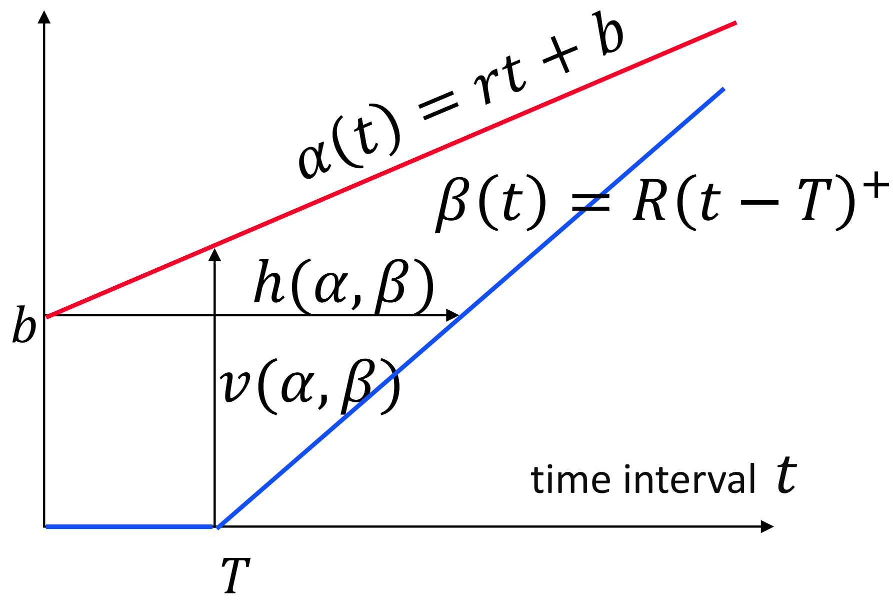



Flow is constrained by arrival curve $\alpha$; served in network element with service curve $\beta$. Then




1. backlog $\le v(\alpha,\beta) = \sup_t \big(\alpha(t) - \beta(t)\big)$
2. if FIFO for this flow, delay $\le h(\alpha,\beta)$
3. output is constrained by arrival curve $\alpha^*(t) = \sup_{u \ge 0} \big(\alpha(t+u) - \beta(u)\big)$
   i.e. $\alpha^* = \alpha \oslash \beta$ (deconvolution)

# Example




One flow, constrained by one token bucket is served in a network element that offers a rate latency service curve

Assume $r \leq R$

Backlog bound: $b + rT$

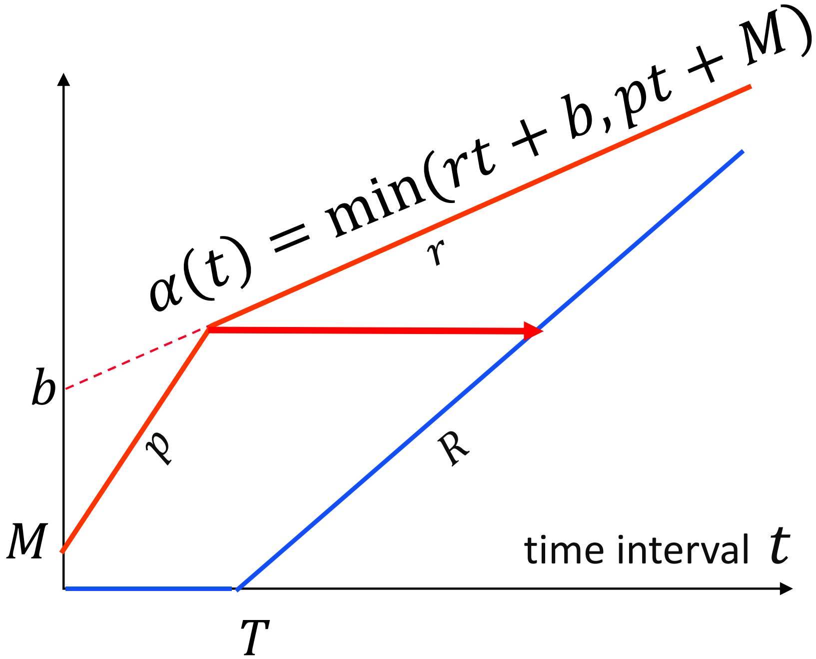
Delay bound: $\frac{b}{R} + T$

Output arrival curve:

$$\alpha^*(t) = rt + b^*$$

with $b^* = b + rT$

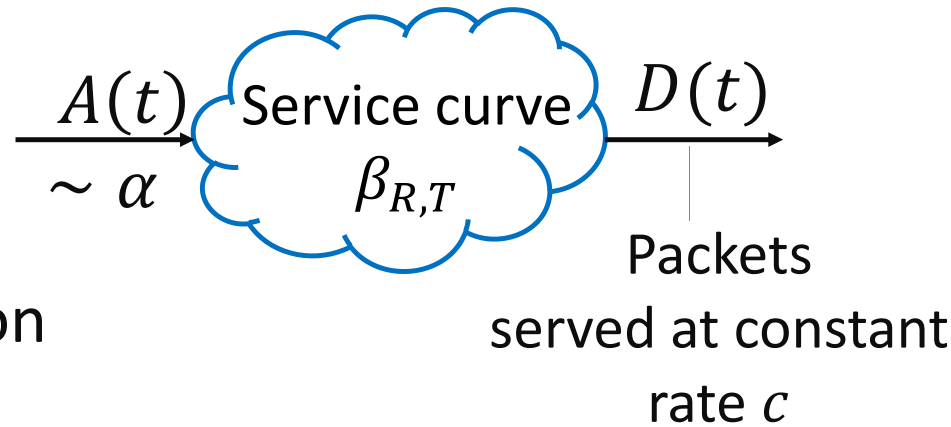
# Example with peak rate limit




Assume $r \leq R$

Delay bound: $\dfrac{M + \frac{b-M}{p-r}(p-R)^{+}}{R} + T$

# An Improved Delay Bound




In many systems we know both

- A service curve characterization e.g. rate latency $\beta_{R,T}$
- Once a packet is selected for transmission, it is served at a constant rate $c$

Improved delay bound is

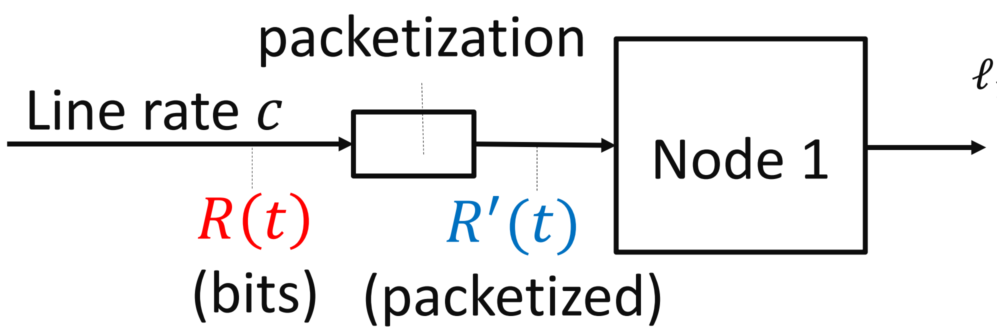
$$\Delta^* = \underbrace{h(\alpha, \beta_{R,T})}_{\text{generic bound}} - \ell_{min}\left(\frac{1}{R} - \frac{1}{c}\right)$$

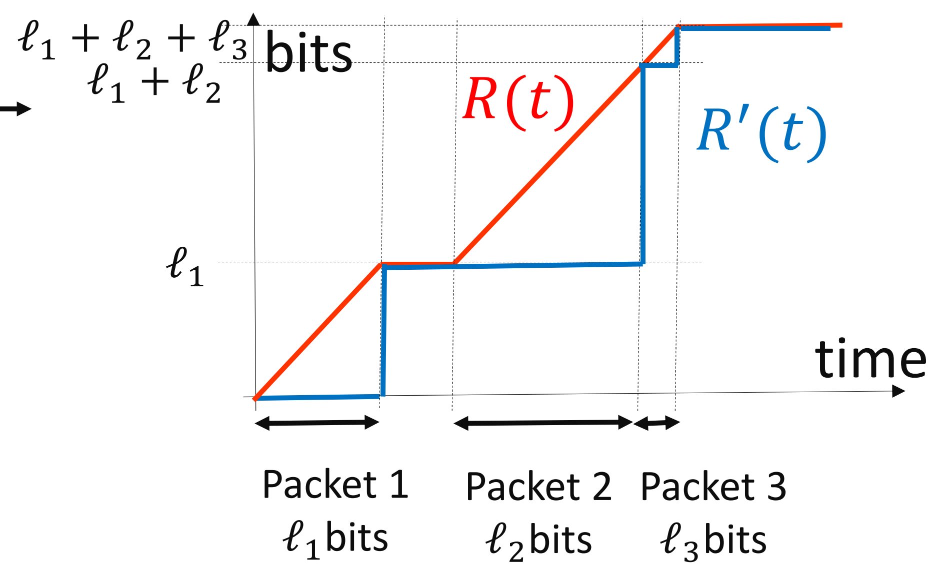
$\ell_{min}$ = lower bound on packet size

[Mohammadpour et al 2019], using max-plus representation of arrival curves

# 4. Packetizer

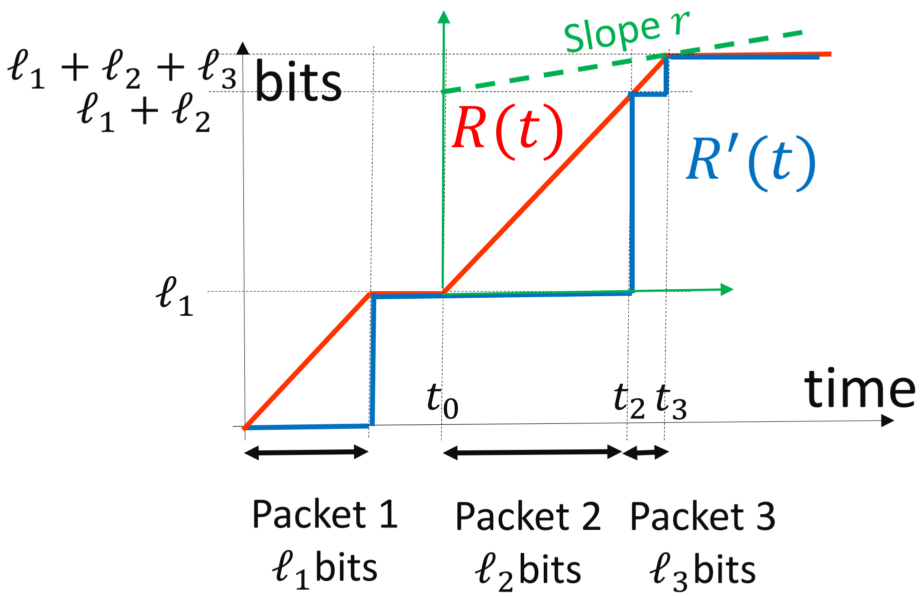
Some nodes receive a packet entirely before processing it (packetization). Typically true for TSN and Detnet. This adds delay and modifies arrival curves.

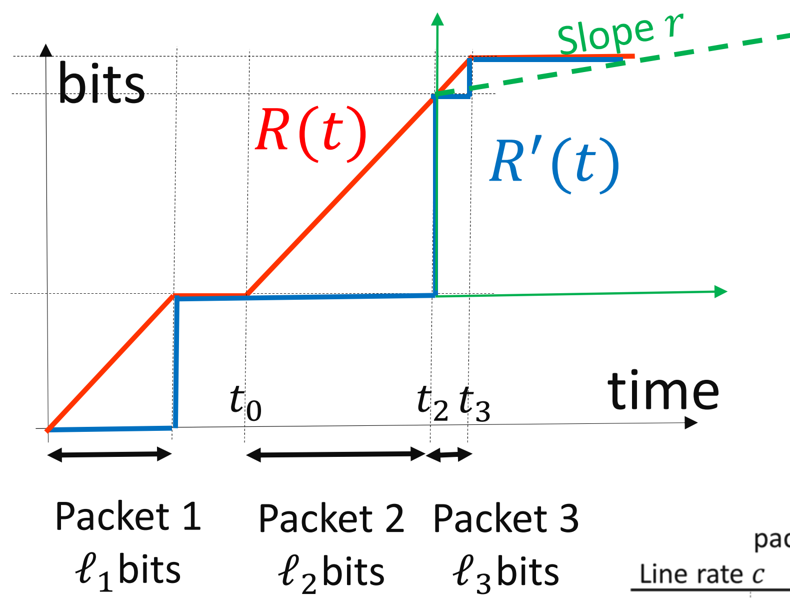


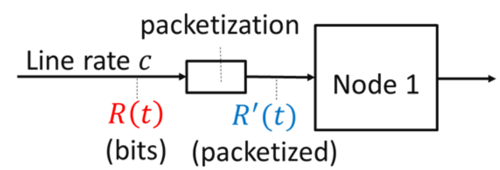




$R$ has arrival curve $\alpha(t) = ct$, obviously $R'$does not.

# Packetization affects arrival curves



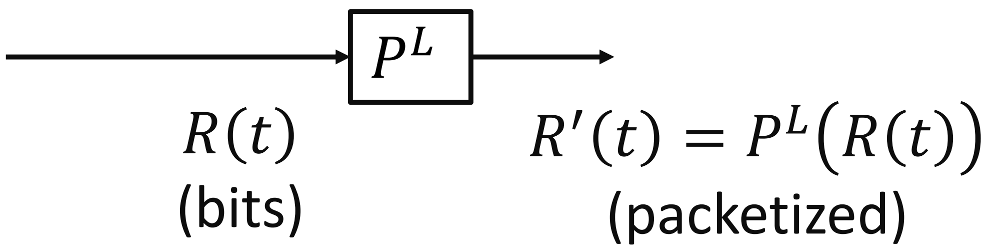




Assume $R$ also has arrival curve $rt + b$. Does the same hold for $R'$ ?

No ! Packets may be accelerated by packetization

$\ell_2 = b, t_2 - t_0 = \frac{b}{c}, \ell_3 = \frac{br}{c},$
$R(t)$ has $rt + b$ as arrival curve but $R'$ does not

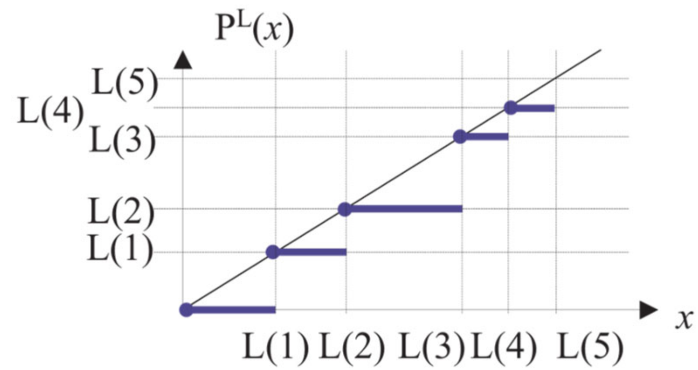
# Packetization is modelled by means of packetizer

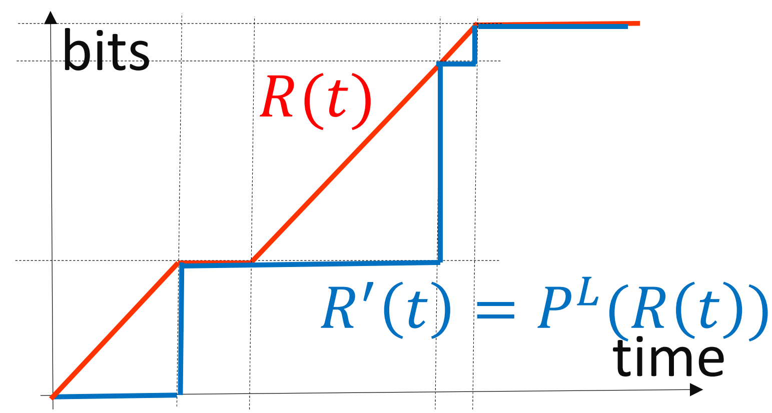



Given a sequence of packet lengths $\ell_1, \ell_2, \ldots$ the function $P^L$ is defined by

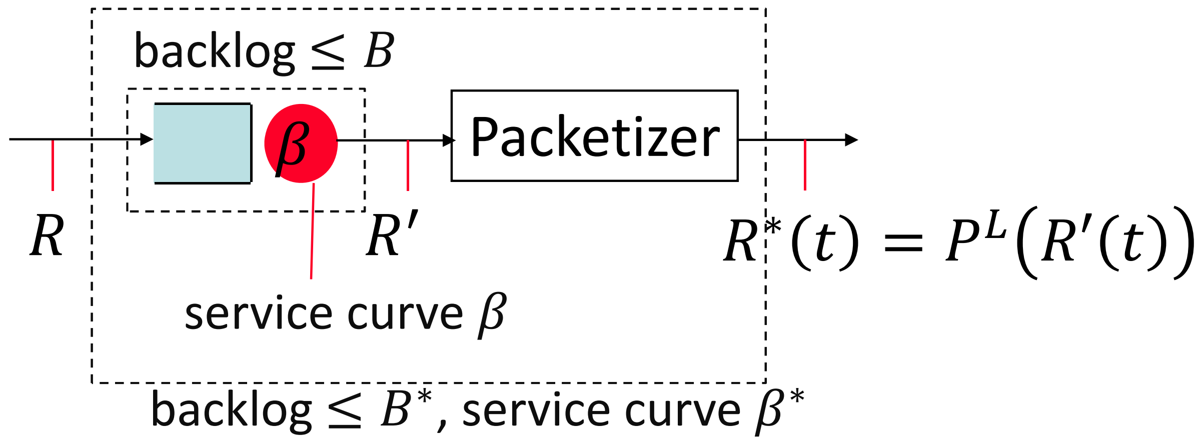
$$P^L(x) = L_n \Leftrightarrow L(n) \leq x < L(n+1)$$

with $L(n) = \ell_1 + \cdots + \ell_n$



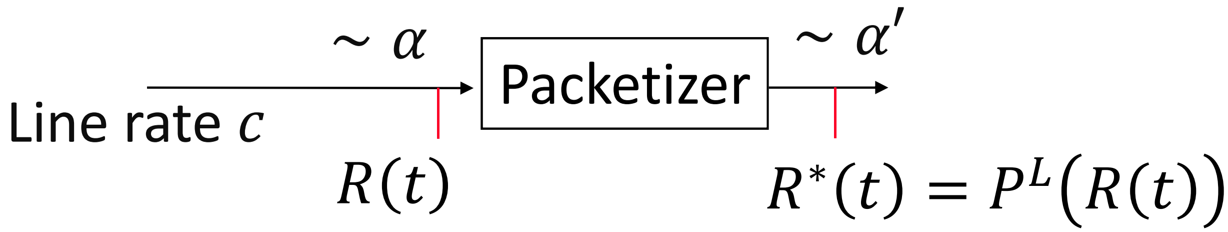
# Effect of Packetization




1. Delay bounds are same for $R'$ and $R^*$
2. Backlog bound $B^* = B + \ell^{max}$
3. Service curve $\beta^*(t) = [\beta(t) - \ell^{max}]^+$

$\ell^{max} = \max_n \ell_n$

# Effect of Packetization (continued)

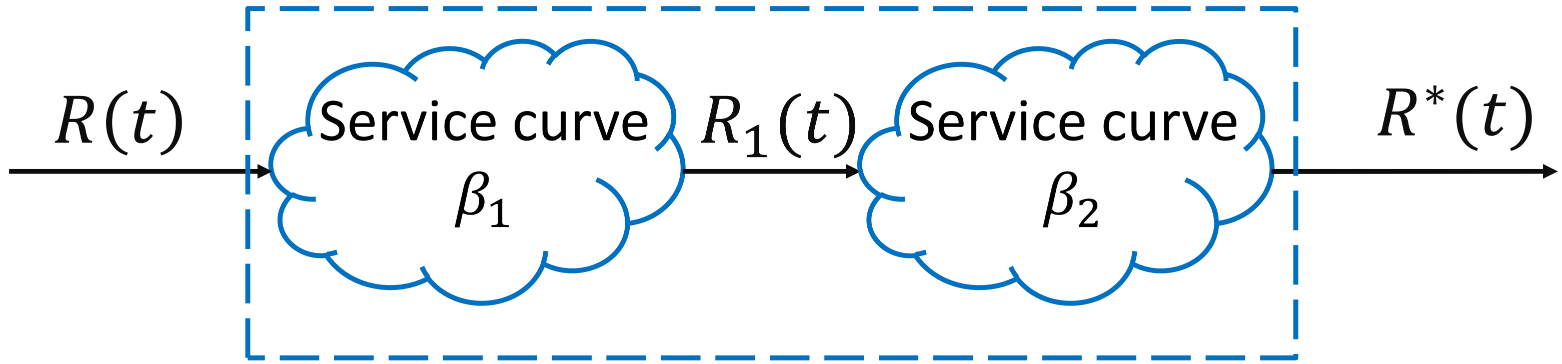



$\ell^{max} = \max_n \ell_n$

4. If a fluid flow $R(t)$ has arrival curve $\alpha(t)$ then the packetized flow $P^L(R(t))$ has arrival curve $\alpha'(t) = \alpha(t) + \ell^{max}$
5. If a fluid flow $R(t)$ has arrival curve $\alpha(t)$ and is received at a constant line rate $c$ then the packetized flow $P^L(R(t))$ has arrival curve $\alpha\left(t + \frac{\ell^{max}}{c}\right)$
   e.g $\alpha(t) = rt + b \Rightarrow \alpha'(t) = rt + b + \frac{r}{c}\ell^{max}$
   [Thomas et al 2019]

See also Packet curves [Bouillard et al, 2011]

# 5. Concatenation

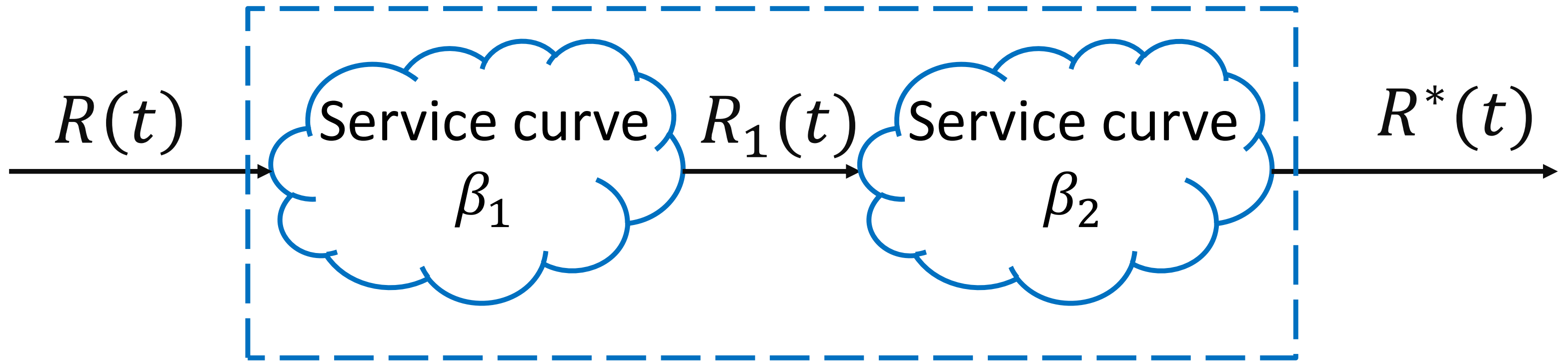



$$\boxed{\beta = \beta_1 \otimes \beta_2}$$

A flow is served in series, network element $i$ offers service curve $\beta_i$.
The concatenation offers to flow the service curve $\beta = \beta_1 \otimes \beta_2$

**Proof:** $R^* \geq R_1 \otimes \beta_2 \geq (R \otimes \beta_1) \otimes \beta_2 = R \otimes (\beta_1 \otimes \beta_2)$

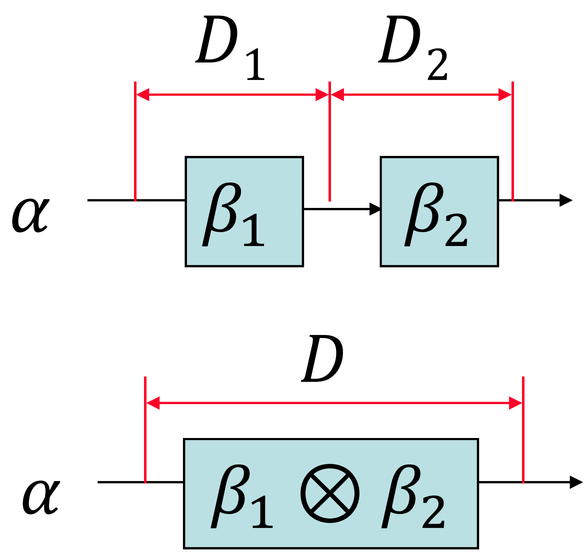
# Example




$$\boxed{\beta = \beta_1 \otimes \beta_2}$$

If $\beta_i$ is rate-latency $R_i, T_i$ then the concatenation $\beta$ is rate-latency $R = \min(R_1, R_2)$ and $T = T_1 + T_2$

# Pay Bursts Only Once




$$\alpha(t) = rt + b$$
$$\beta_1(t) = R(t - T_1)^+$$
$$\beta_2(t) = R(t - T_2)^+$$
$$r \leq R$$

one flow constrained *at source* by $\alpha$

end-to-end delay bound computed *node-by-node* (also accounting for increased burstiness at node 2):

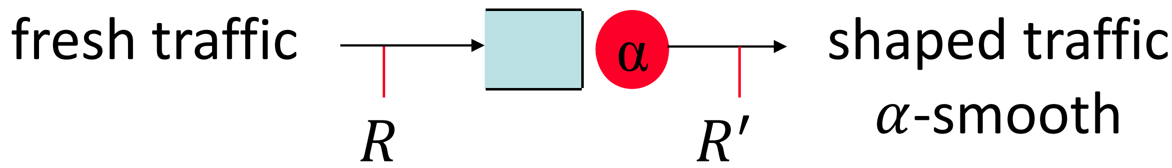
$$D_1 + D_2 = \frac{2b + rT_1}{R} + T_1 + T_2$$

computed *by concatenation:*

$$D = \frac{b}{R} + T_1 + T_2$$

i.e. worst cases cannot happen simultaneously – concatenation captures this !

# 6. Shapers




(Fluid) Shaper forces output to be constrained by arrival curve $\alpha$

(Fluid) Greedy Shaper stores data in a buffer only if needed

Examples:

constant bit rate link is fluid greedy shaper for $\alpha(t) = ct$

Shaper constraints are

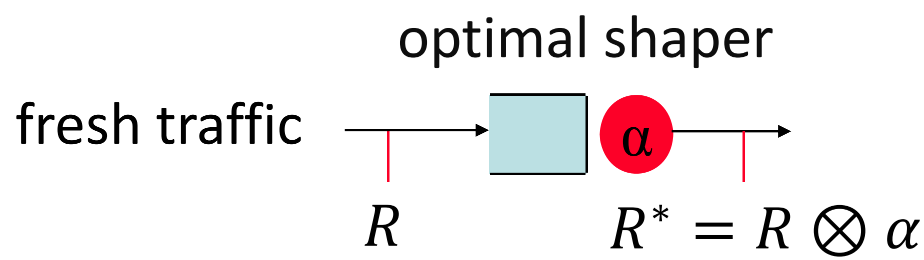
$$R'(t) \leq R(t)$$
$$R'(t) \leq (R' \otimes \alpha)(t)$$

# I/O Characterization of Fluid Greedy Shaper

Min-Plus Residuation Theory [Baccelli et al. 1992] ⇒

The problem
(where the unknown is
the function $R'$)

$$R'(t) \le R(t)$$
$$R'(t) \le (R' \otimes \alpha)(t)$$

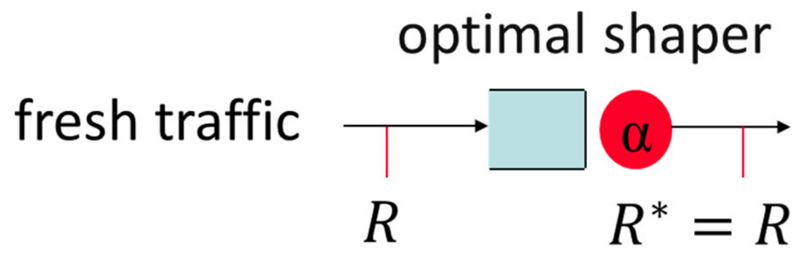



has one maximal solution $R^*$, given by $R^* = R \otimes \bar{\alpha}$

We can always assume that $\alpha$ is sub-additive and $\alpha(0) = 0$ so that $R^* = R \otimes \alpha$

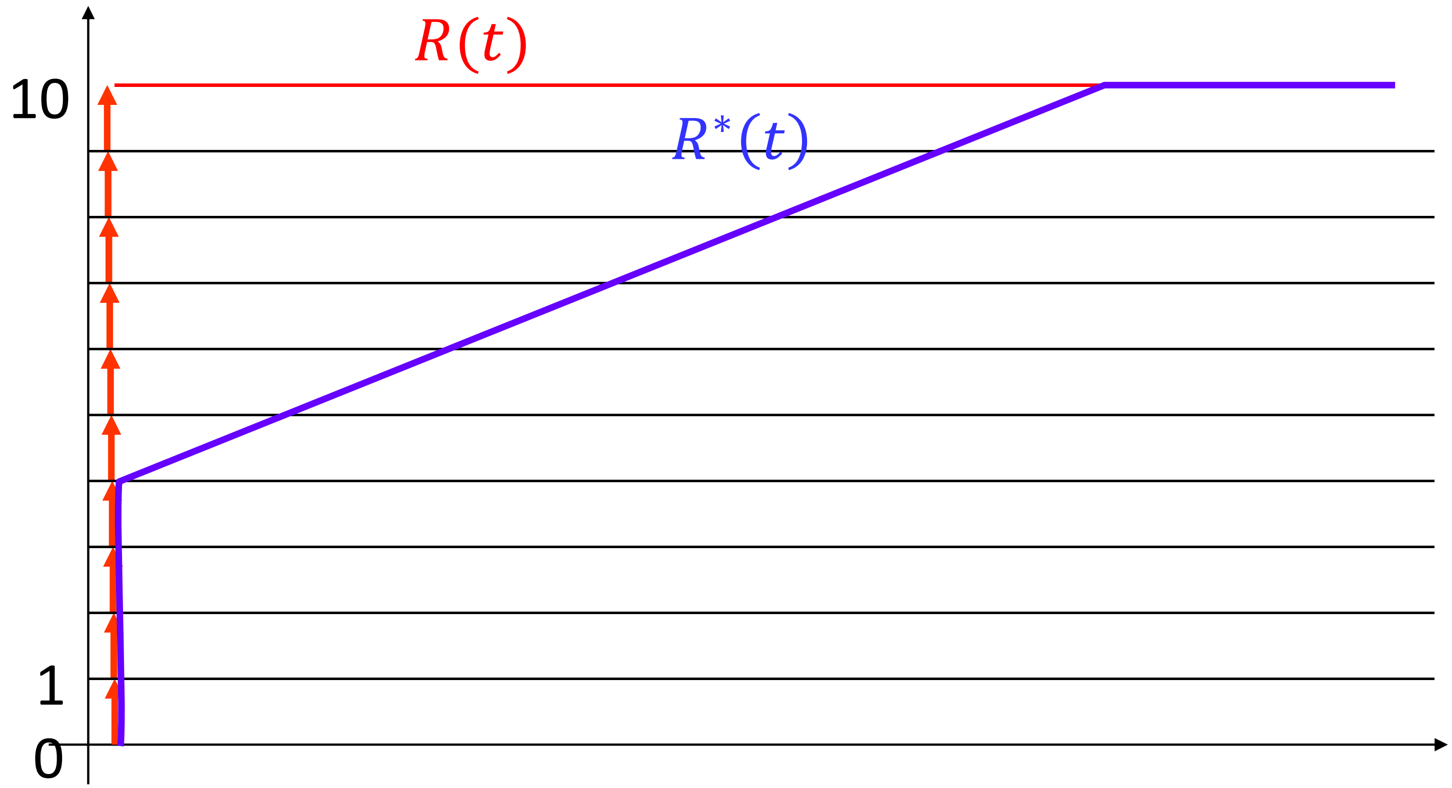
⇒ greedy shaper has service curve $\alpha$

# Example



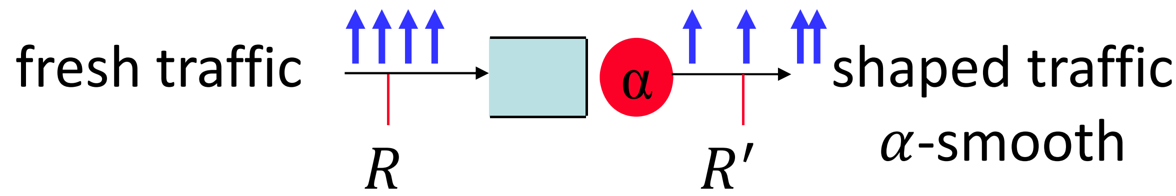

$$R^* = R \otimes \alpha$$

$$\alpha(t) = rt + b$$




$b = 4\,p.u.$

# Packetized Shapers




(Packetized) Shaper

forces output to be packetized and constrained by arrival curve $\alpha$

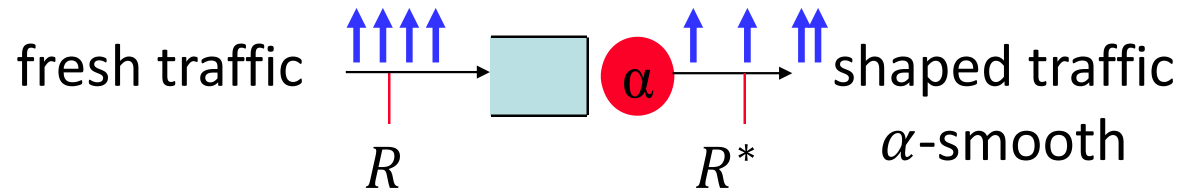
(Packetized) Greedy Shaper stores packets in a buffer only if needed

i.e. delivers the maximal solution to

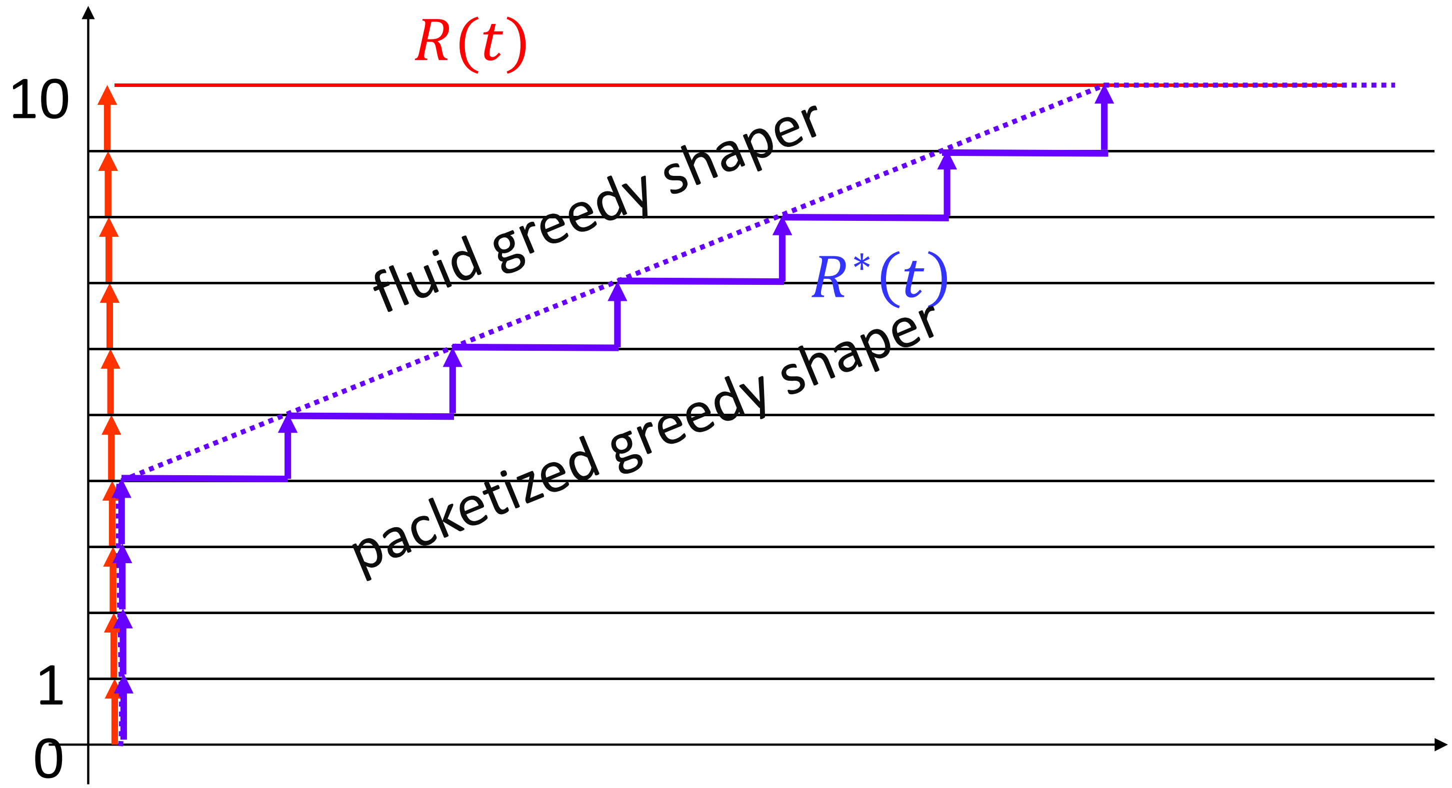
$$R'(t) \leq R(t)$$
$$R'(t) \leq (R' \otimes \alpha)(t)$$
$$R'(t) \leq P^L(R'(t))$$

Min-plus residuation ⇒ Existence of a maximal solution

# Example

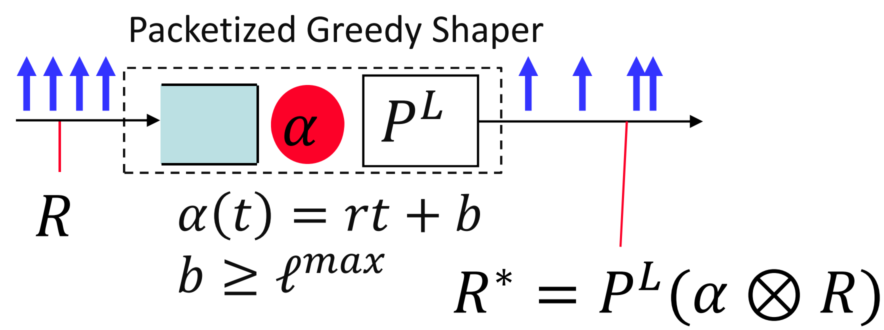



$$\alpha(t) = rt + b$$




$r = 4\ p.u.$

# I/O Characterization



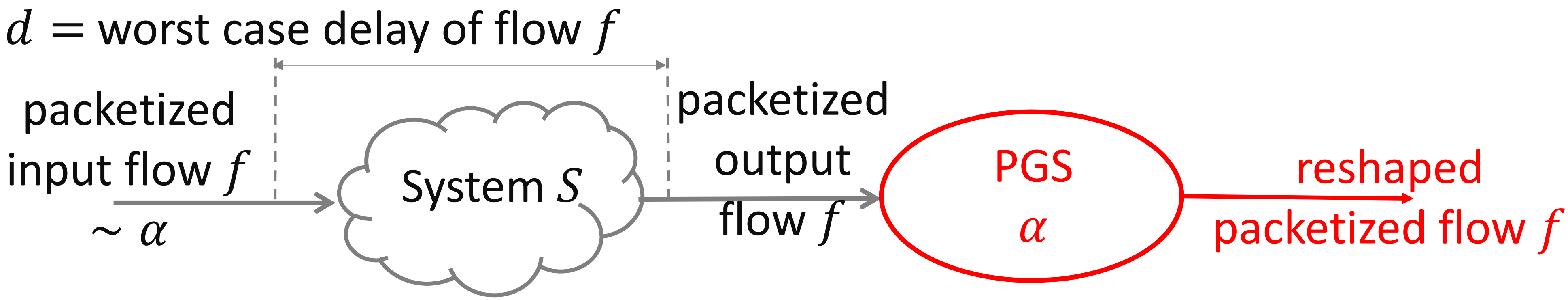

If (C) $\alpha$ is piecewise-linear, concave and $\alpha(0^+) \geq \ell^{max}$ then $R^* = P^L(\alpha \otimes R)$
i.e. packetized greedy shaper = fluid greedy shaper + packetizer.

(if (C) does not hold this may not be true)

Example: Linux Token Bucket filter TBF$(r, b)$ is the packetized greedy shaper for $\alpha(t) = rt + b$.

[Le Boudec 2002]

# Re-Shaping Does Not Increase Worst-Case Delay

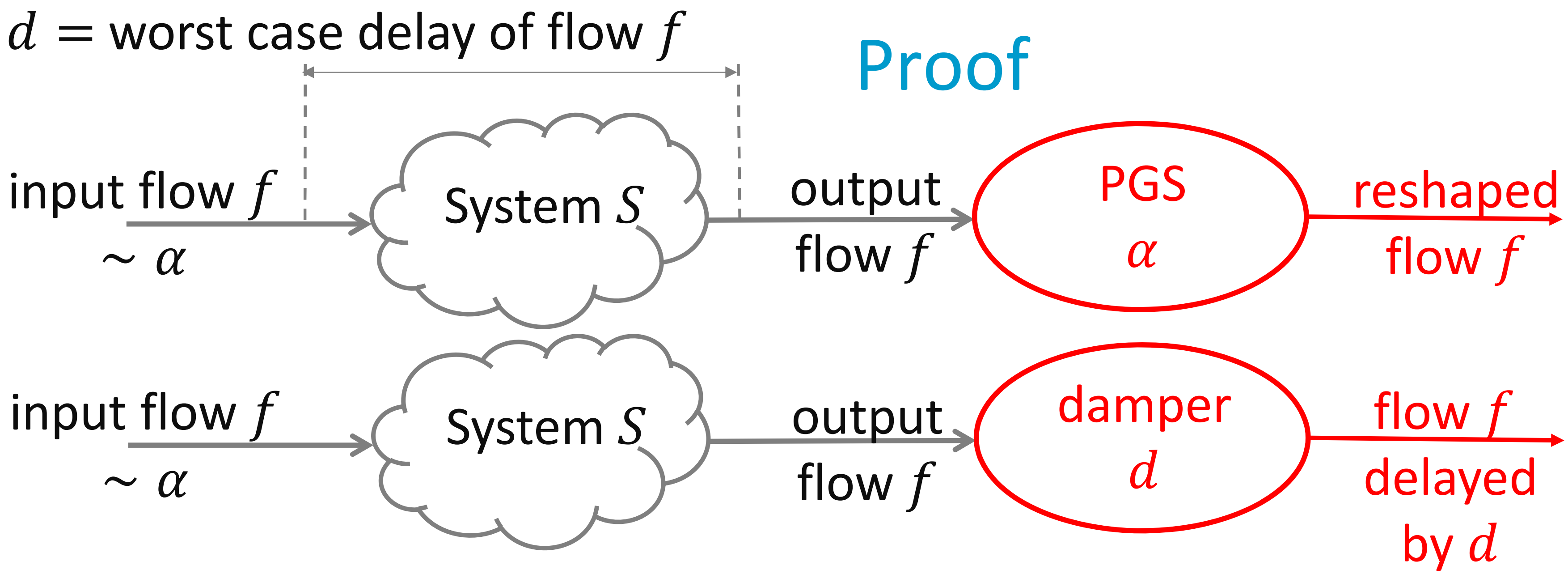



One packetized flow $f$ goes through a system $S$; system $S$ is FIFO for flow $f$; flow $f$ is constrained by arrival curve $\alpha$ at input to $S$; output flow $f$ is reshaped at output through a packetized greedy shaper for same arrival curve $\alpha$ (shaper is FIFO)

**Theorem:** The worst case delay of flow $f$ is not increased

Re-shaping is for free ! [Le Boudec 2018] -- True whether (C) holds or not.

# Proof




Replace packetized greedy shaper by **damper** [Verma et al 1991]:
Damper forces total delay of flow $f$ to be exactly $d$; Damper is causal if $d$ is $\geq$ worst-case delay through $S$.

Output of damper is input flow $f$, time-shifted by $d \Rightarrow$ is $\alpha-$smooth $\Rightarrow$ Damper is a packetized shaper $\Rightarrow$ (maximal property of packetized greedy shaper) flow $f$ delayed by $d$ is no earlier than reshaped flow $f$

# 7. Outlook

Research themes

interleaved shapers
use of re-shaping in large scale deterministic networks

tight latency bounds
stateless shapers

latency lower bounds

tight bounds accounting for real-time application layer
results that combine min-plus and max-plus reps

packet curves

...

# Tools

- The DiscoDNC is an academic Java implementation of the network calculus framework.[6]
- The RTC Toolbox is an academic Java/MATLAB implementation of the Real-Time calculus framework, a theory quasi equivalent to network calculus.[4]
- The CyNC [7] tool is an academic MATLAB/Symulink toolbox, based on top of the RTC Toolbox. The tool was developed in 2004-2008 and it is currently used for teaching at Aalborg university.
- The RTaW-PEGASE is an industrial tool devoted to timing analysis tool of switched Ethernet network (AFDX, industrial and automotive Ethernet), based on network calculus.[8]
- The Network calculus interpreter is an on-line (min,+) interpreter.
- The WOPANets is an academic tool combining network calculus based analysis and optimization analysis.[9]
- The DelayLyzer is an industrial tool designed to compute bounds for Profinet networks.[10]
- DEBORAH is an academic tool devoted to FIFO networks.[11]
- NetCalBounds is an academic tool devoted to blind & FIFO tandem networks.[12][13]
- NCBounds is a network calculus tool in Python, published under BSD 3-Clause License. It considers rate-latency servers and token-bucket arrival curves. It handles any topology, including cyclic ones[14].
- The Siemens Network Planner (SINETPLAN) uses network calculus (among other methods) to help the design of a PROFINET network.[15]

sampled on 2019 Oct 1 from https://en.wikipedia.org/wiki/Network_calculus

# Conclusion

Network calculus main concepts:

cumulative functions (time domain)
arrival curve, minimal service curve (time interval domain)
shapers, concatenation

For packet-based systems and for fluid systems.

Fundamental tools for everyone in real-time networks and systems!

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