Understanding The Simulation Of Mobility Models with Palm Calculus

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Resources

- Random trip model web page:

  http://ica1www.epfl.ch/RandomTrip

  – Links to slides, papers, perfect simulation software

- This tutorial is mainly based on:

  [LV06] The Random Trip Model: Stability, Stationary Regime, and Perfect Simulation, J.-Y. Le Boudec and Milan Vojnović, ACM/IEEE Trans. on Networking, Dec 06

  – Extended journal version of IEEE Infocom 2005 paper
  – Technical report with proofs: MSR-TR-2006-26

  [L04] Understanding the simulation of mobility models with Palm calculus, J.-Y. Le Boudec, Performance Evaluation, 2007

Outline

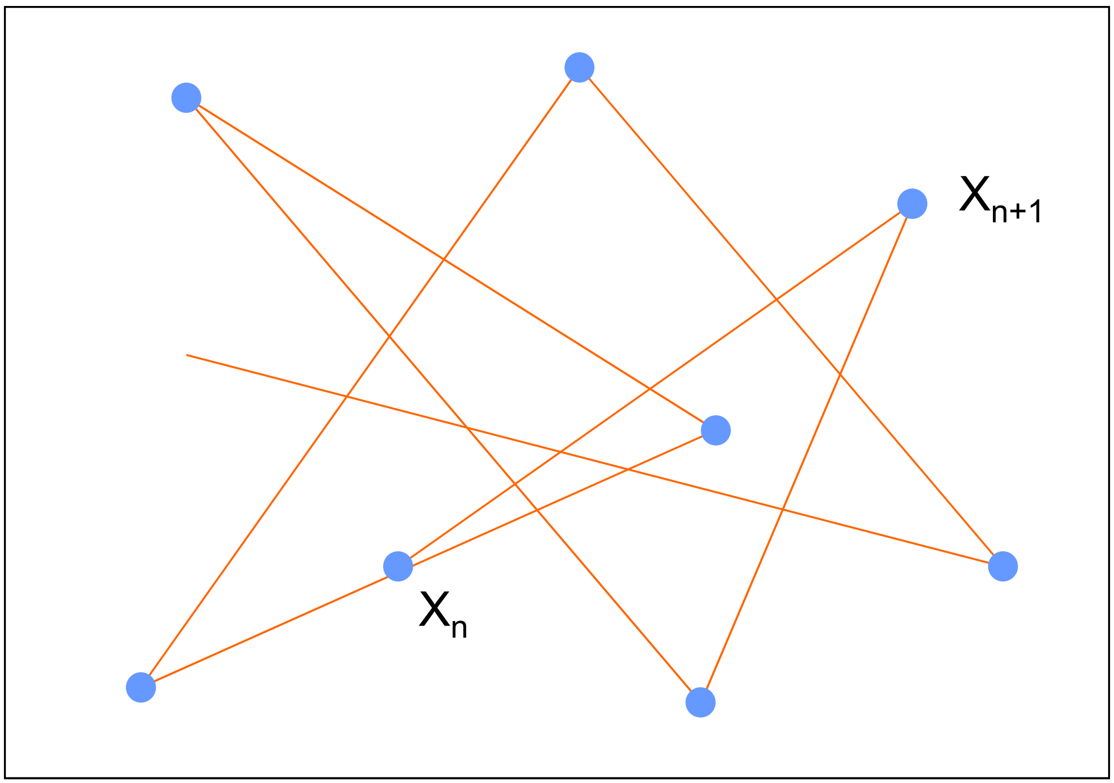
- Simulation Issues with mobility models
- Random trip basic constructs
- Palm calculus instant primer
- Stability condition for random trip
- Time-stationary distributions
- Perfect simulation

Appendix

- FAQ
- Harris recurrence

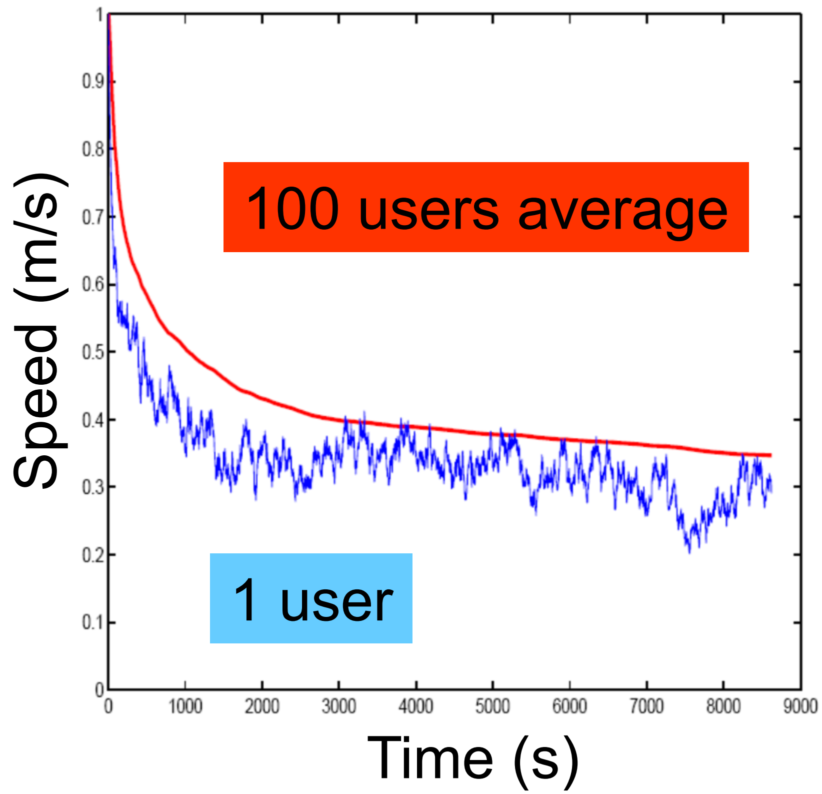
Simplest example: random waypoint (Johnson and Maltz`96)

- Node:
  - Picks next waypoint X_{n+1} uniformly in area
  - Picks speed V_n uniformly in $[v_{min}, v_{max}]$
  - Moves to X_{n+1} with speed V_n



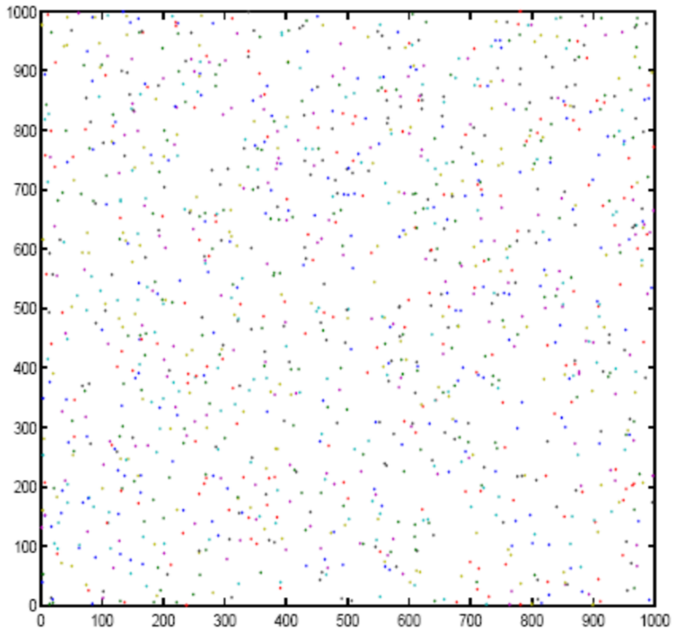
Already the simple model exhibits issues

- Distributions of node speed, position, distances, etc change with time
  - Node speed:

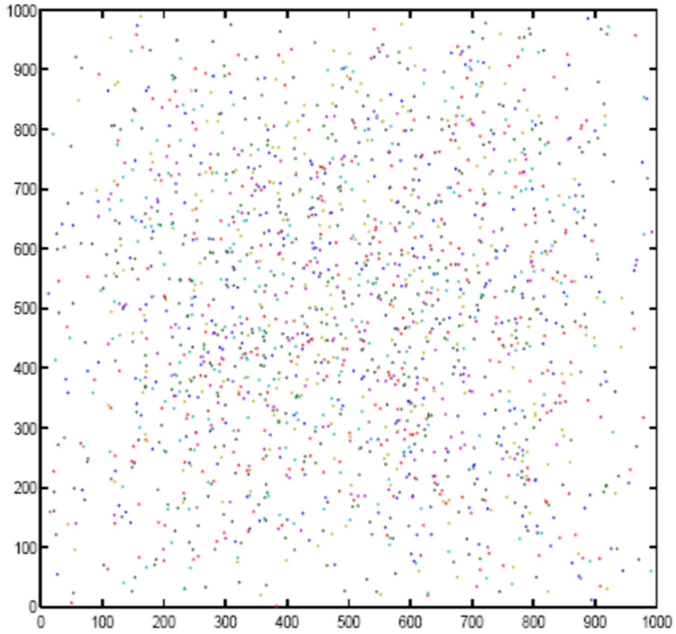


Already the simple model exhibits issues (2)

- Distributions of node speed, position, distances, etc change with time
  - Distribution of node position:



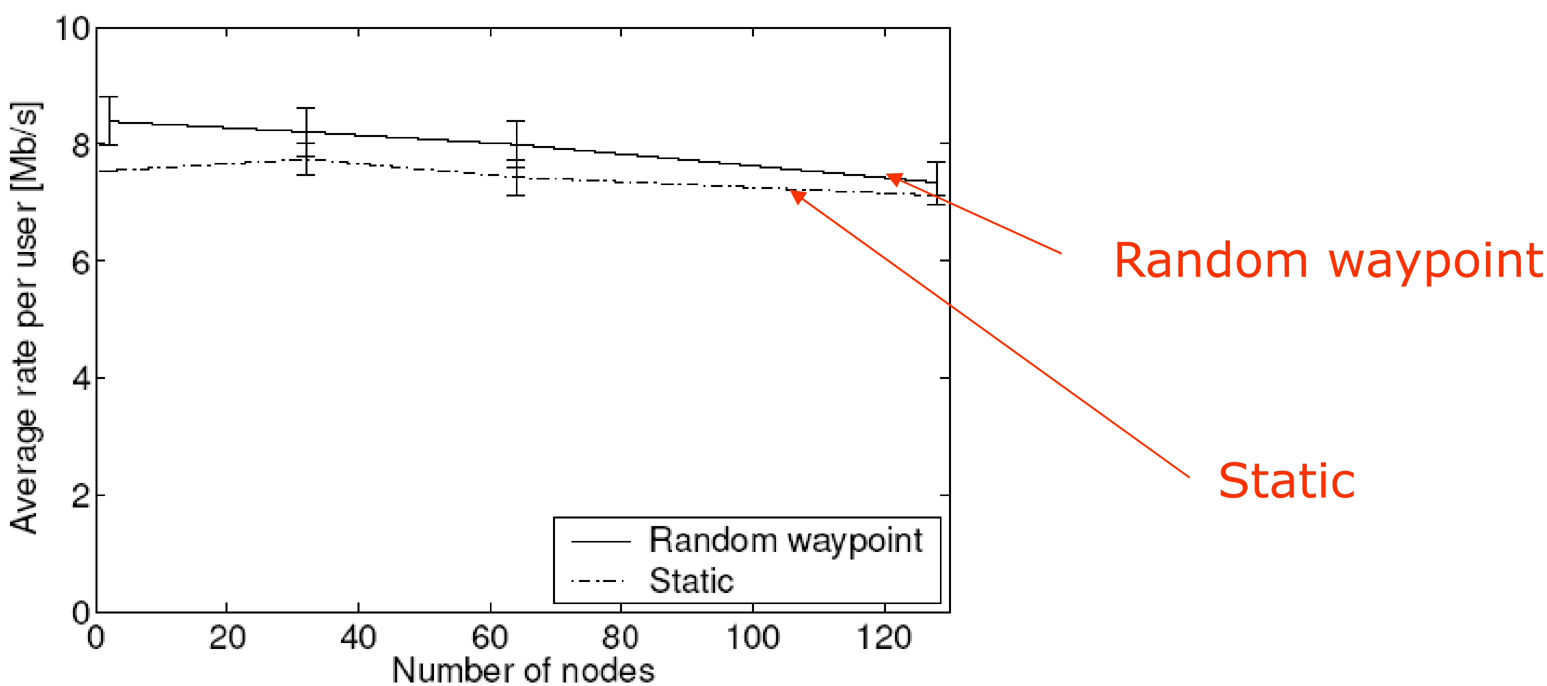
Time = 0 sec



Time = 2000 sec

Why does it matter ?

- A (true) example: Compare impact of mobility on a protocol:
  - Experimenter places nodes uniformly for static case, according to random waypoint for mobile case
  - Finds that static is better
- **Q.** Find the bug !



- **A.** In the mobile case, the nodes are more often towards the center, distance between nodes is shorter, performance is better
- The comparison is flawed. Should use for static case the same distribution of node location as random waypoint. *Is there such a distribution to compare against ?*

Issues with Mobility Models

- Is there a stable distribution of the simulation state (*time-stationary distribution*), reached if we run the simulation long enough ?
- If so:
  - How long is long enough ?
  - If it is too long, is there a way to get to the stable distribution without running long simulations *(perfect simulation)* ?

The Random trip model

- A broad model of independent node movements
  - Including RWP, realistic city maps, etc
- Defined by a set of conditions on trip selection
- Conditions ensure issues mentioned above are under control
  - Model stability (defined later)
  - Model permits perfect simulation
    - Algorithm in this slide deck
    - Perfect simulation = distribution of node mobility is time-stationary throughout a simulation

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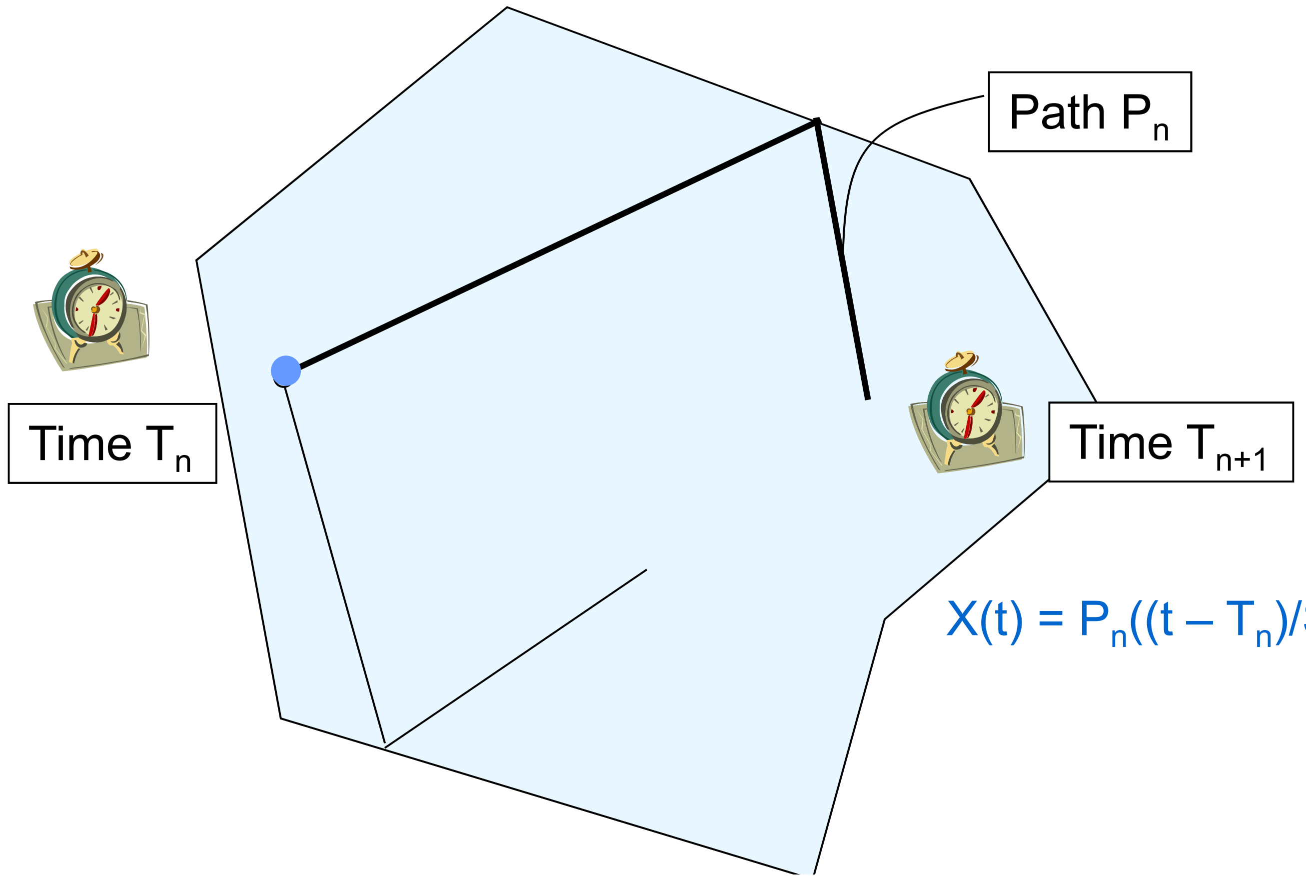
Random trip basic constructs
» Outline «

- Initially: a mobile picks a trip, i.e. a combination of 3 elements
  - A path in a catalogue of paths
  - A duration
  - A phase

- A end of trip, mobile picks a new trip
  - Using a trip selection rule
  - Information required to sample next trip is entirely contained in path and phase of the trip that just finished (*Markov* property)

Illustration of basic constructs

- At end of *(n-1)st* trip, at time T_n, mobile picks
  - Path P_n
  - Duration $S_n = T_{n+1} - T_n$
  - (also a phase – see later)
  - This implicitly defines speed and location X(t) at t 5 $[T_n, T_{n+1}]$



$$X(t) = P_n((t - T_n)/S_n),\ T_n \leq t < T_{n+1}$$

Random waypoint is a random trip model

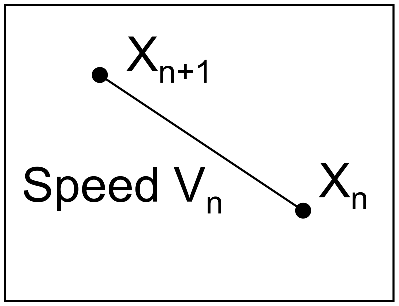
- (Assume in this slide model without pause)
- At end of trip *n-1*, mobile is at location X_n
  - Sample location X_{n+1} uniformly in area

    Path P_n is shortest path from X_n to $X_{n_{+1}}$

    $P_n(u) = (1 - u)\, X_n + u\, X_{n+1}$ for u 5 [0,1]

  - Sample numerical speed V_n ¿ 0 from a given speed distribution

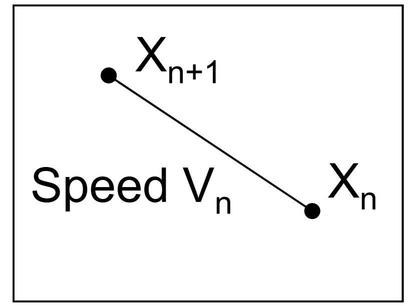
    This defines duration:
    $S_n = ||X_{n+1} - X_n|| \,/\, V_n$
- (Markov property): Information required to sample next trip (location X_n) is entirely contained in path and phase of previous trip



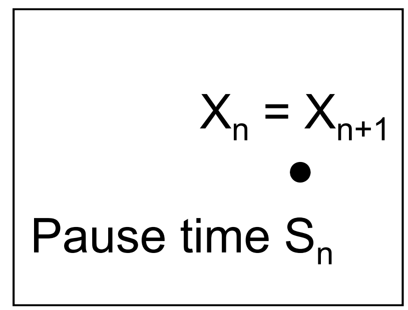
Random waypoint *with pauses* is a random trip model

- Phase I_n is either move or pause
- At end of trip *n-1*:

  If phase I_{n-1} was pause then
  - I_n = move (next trip is a move)
  - Sample X_{n+1} and V_n as on previous slide



  Else
  - I_n = pause (next trip is a move)
  - Path: $P_n(u) = X_n$ for u 5[0,1]
  - Pick S_n from a given pause time distribution



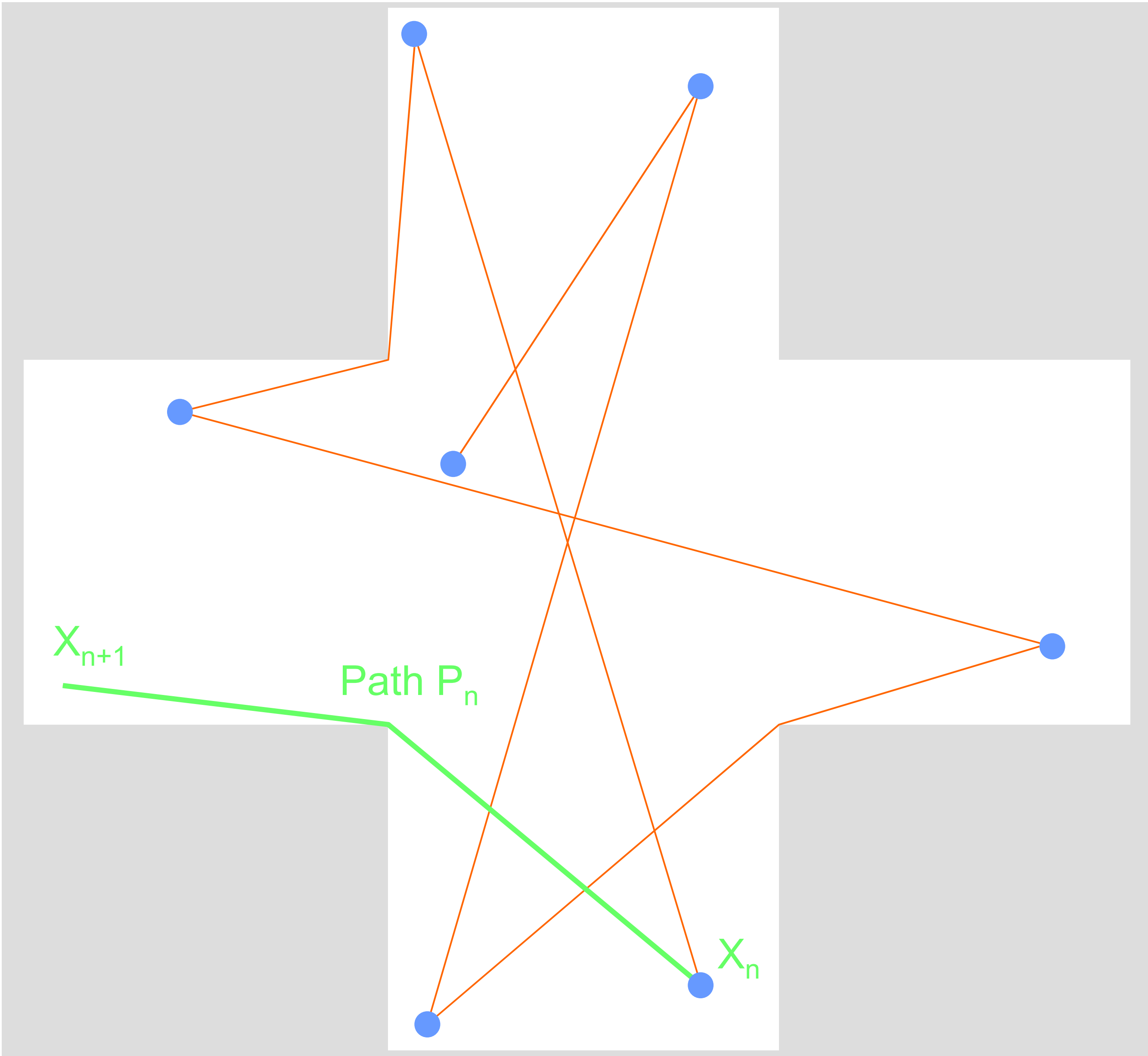
- (Markov property): Information required to sample next trip (phase I_n, location X_n) is entirely contained in path and phase of previous trip

Catalogue of examples

- Random waypoint on general connected domains
  - Swiss Flag
  - City-section
- Restricted random waypoint
  - Inter-city
  - Space-graph
- Random walk on torus
- Billiards
- Stochastic billiards

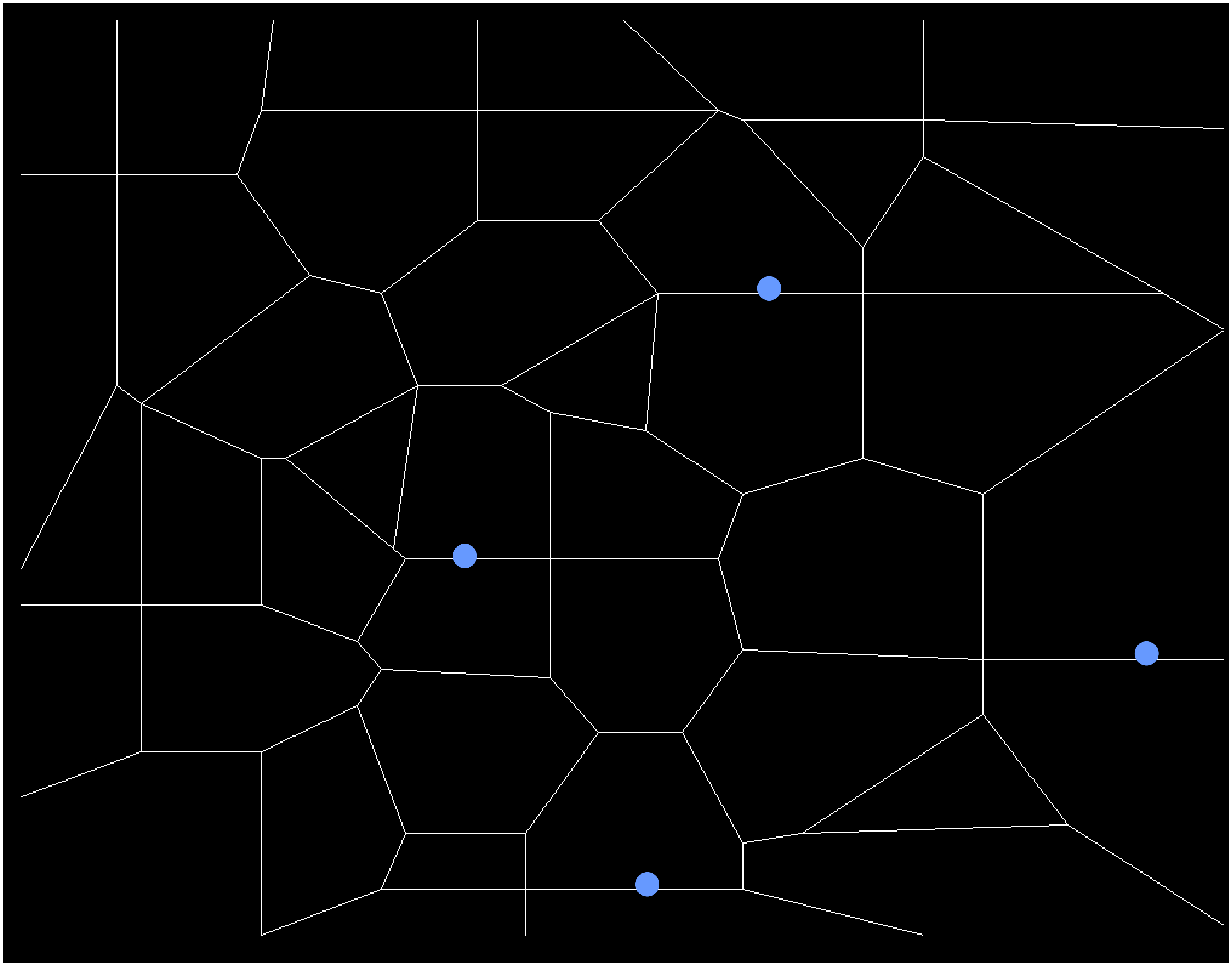
Random waypoint on general connected domain

- Swiss Flag [LV05]
- Non convex domain



Random waypoint on general connected domain (2)

- City-section, Camp et al [CBD02]



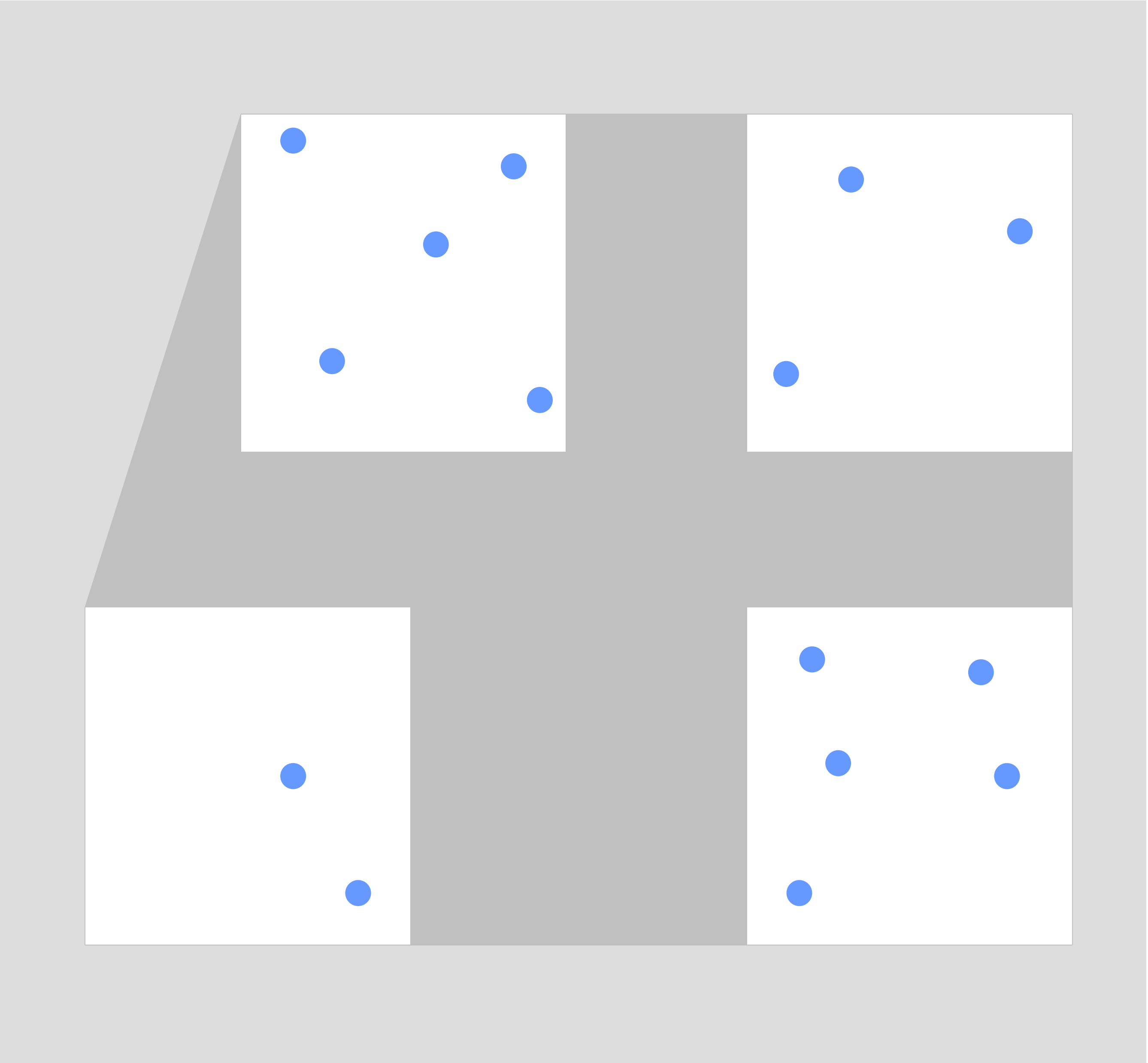
Restricted random waypoint

- Inter-city, Blazevic et al [BGL04]
- Stay in one subdomain for some time then move to other

Here phase is
(I_n, L_n, L_{n+1}, R_n)

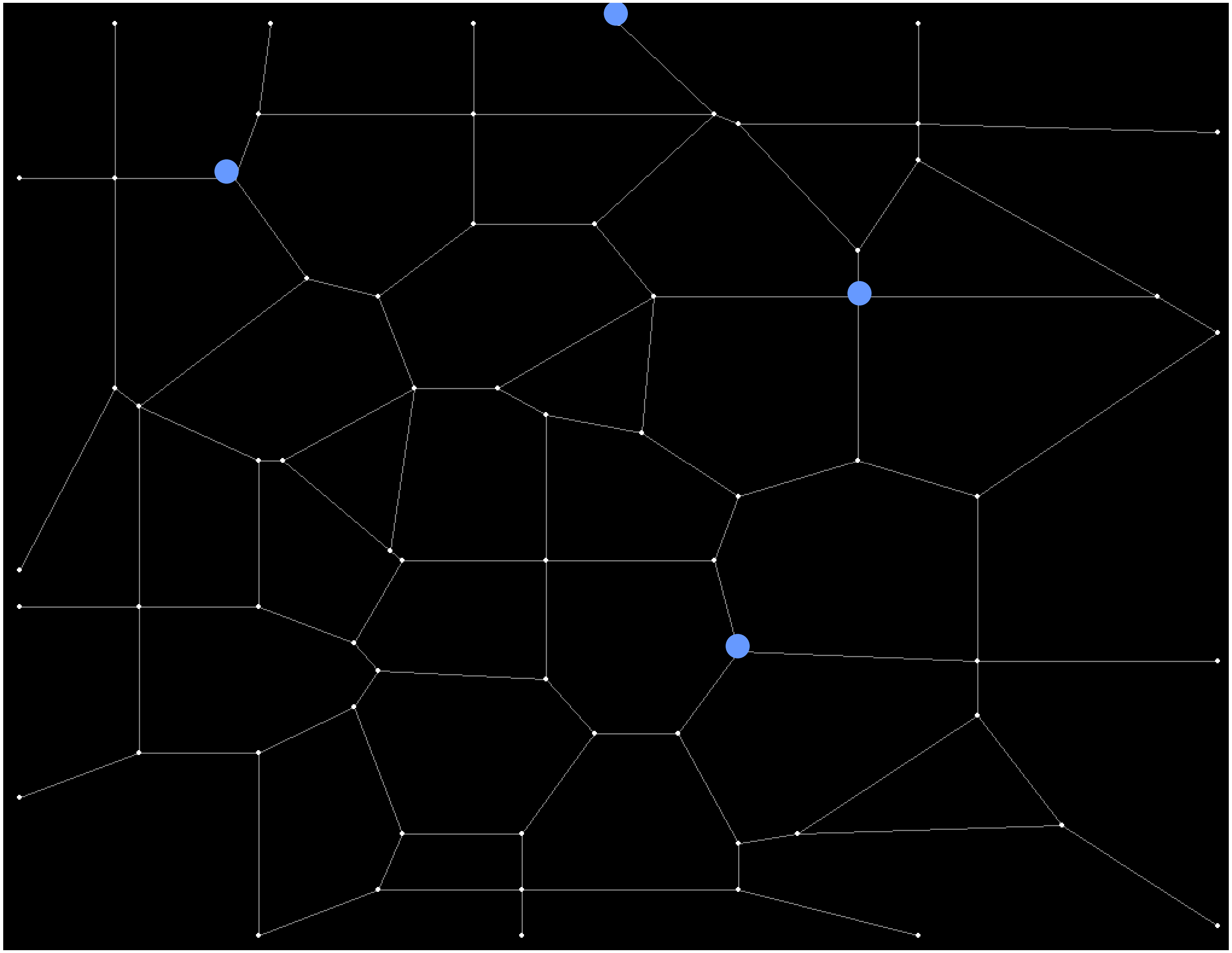
where

I_n = pause or move
L_n = current sub-domain
L_{n+1} = next subdomain
R_n = number of trips in this visit to the current domain



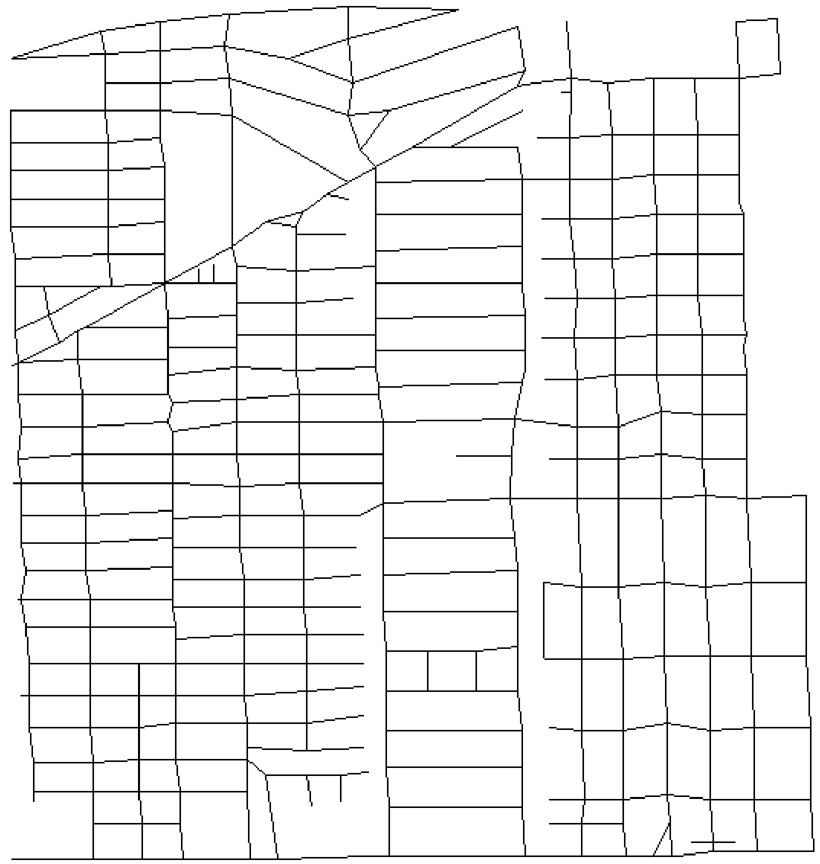
Restricted random waypoint (2)

- Space-graph, Jardosh et al, ACM Mobicom 03 [JBAS03]



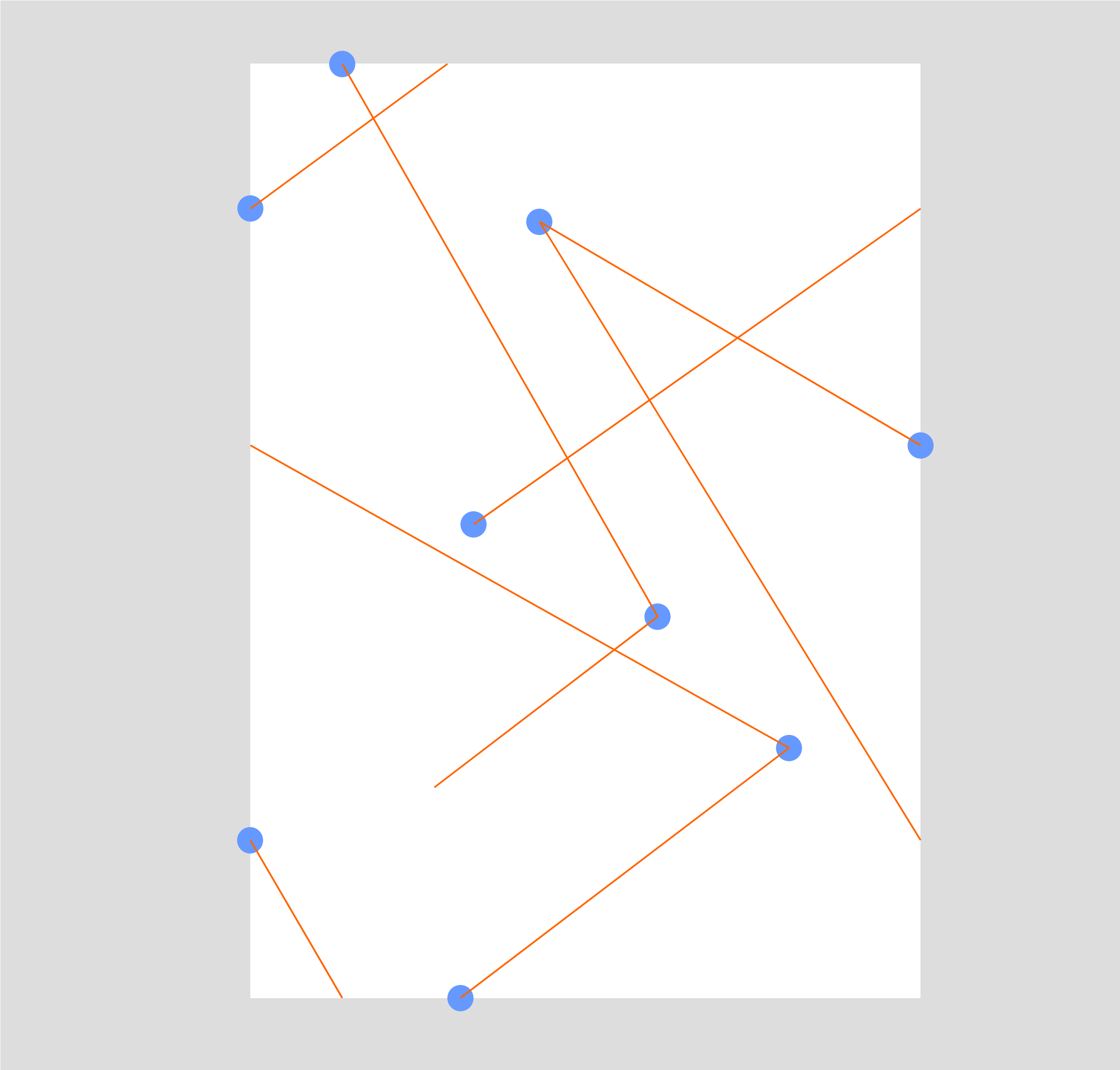
Road maps available from road-map databases

- Ex. US Bureau's TIGER database
  - Houston section
  - Used by PalChaudhuri et al [PLV05]



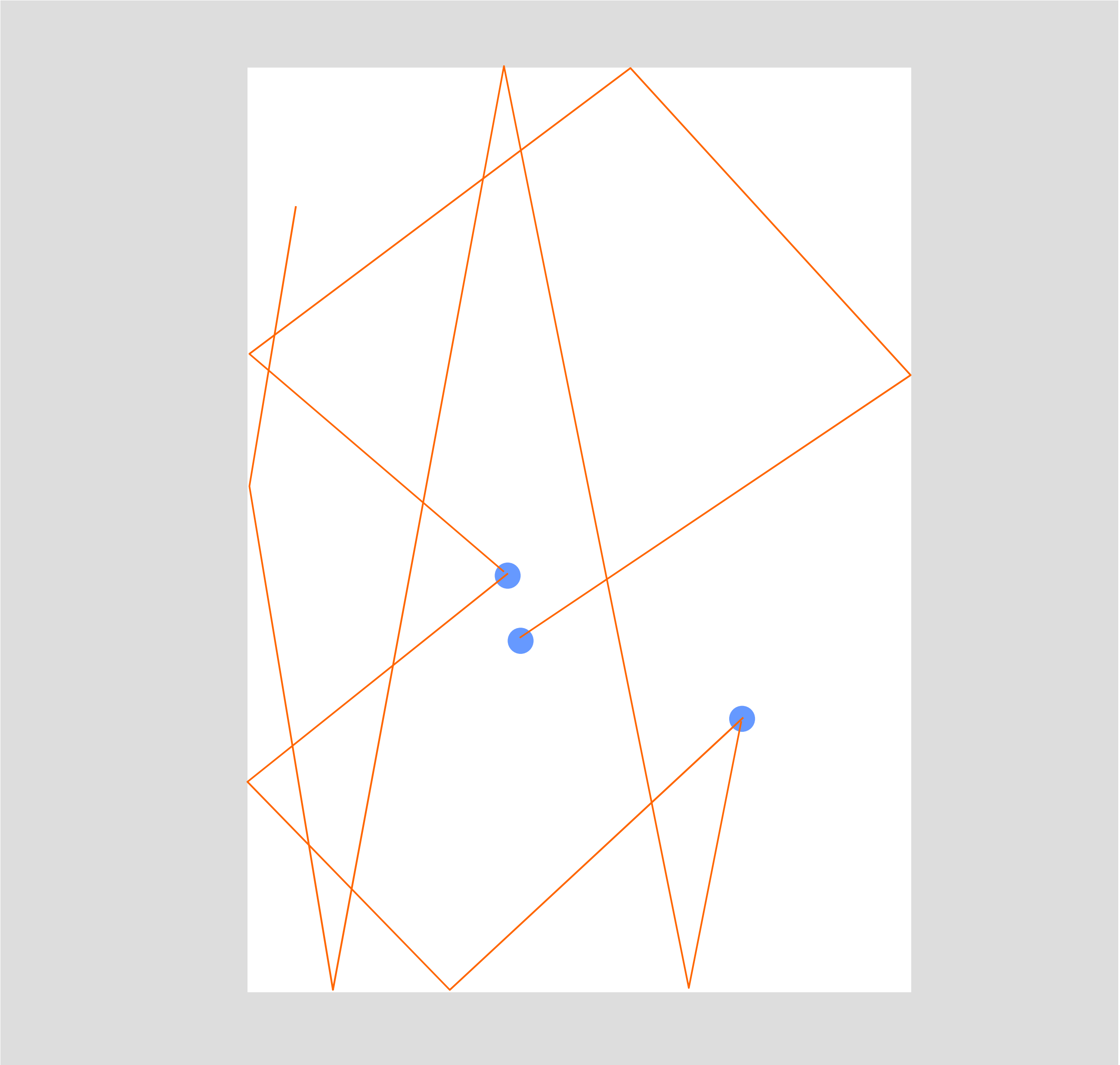
Random walk on torus

- [LV05]
- a.k.a. random direction with wrap around (Nain et al [NT+05])



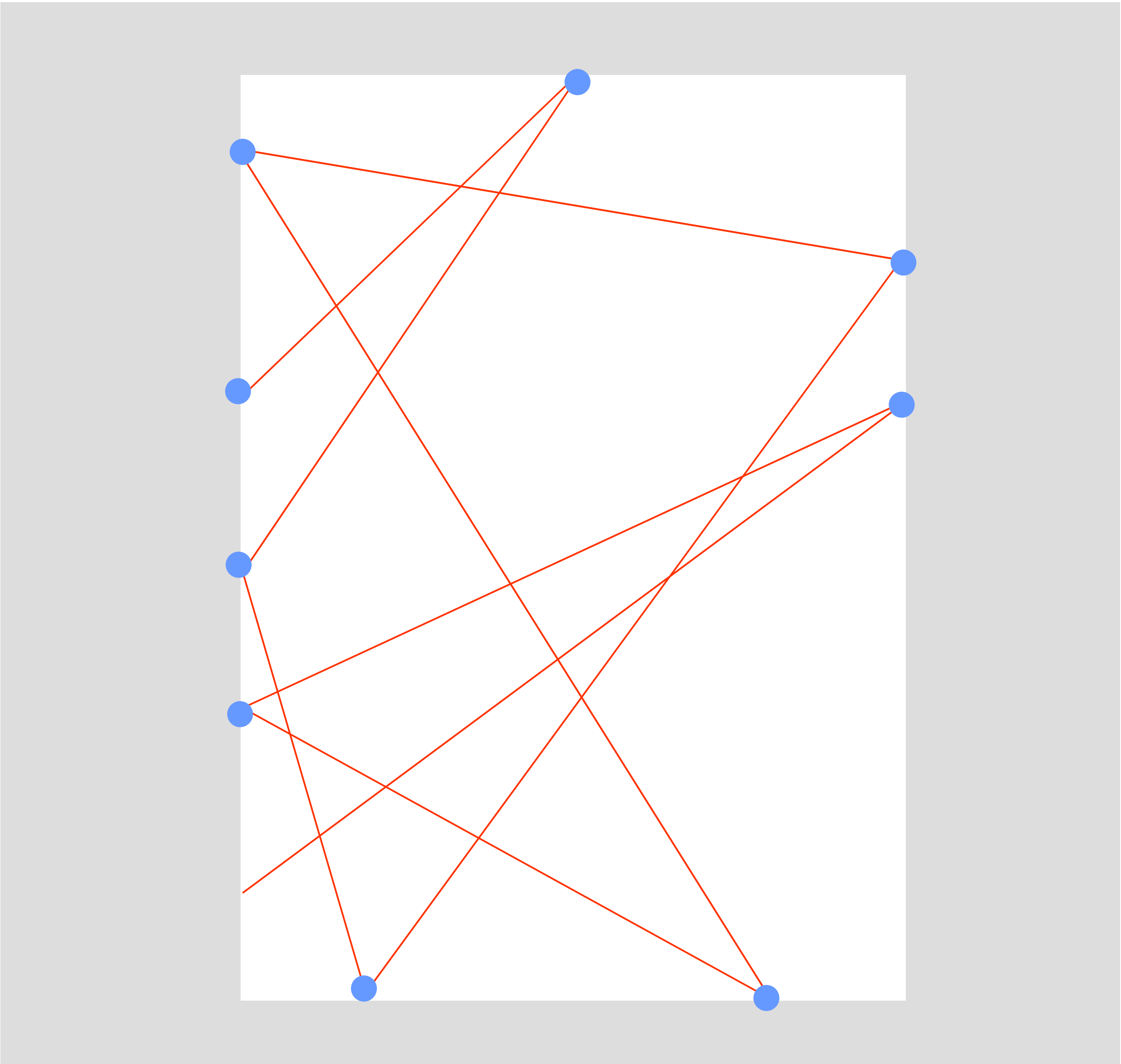
Billiards

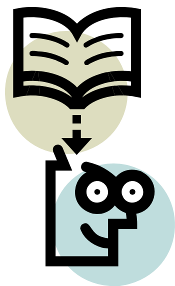
- [LV05]
- a.k.a. random direction with reflection (Nain et al [NT+05])



Stochastic billiards

- Random direction model, Royer et al [RMM01]
- See also survey [CBD02]





Random trip basic constructs
» Summary «

- Trip is defined by phase, path, and duration
- The abstraction accommodates many examples
  - Random waypoint on general connected domains
  - Random walk with wrap around
  - Billiards
  - Stochastic billiards

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Palm Calculus

- Relates time averages versus event averages
  - An old topic in queueing theory
  - Now well understood by mathematicians under the name Palm Calculus

Palm Calculus Framework

- A stationary process (simulation) with state S_t.
- Some quantity X_t measured at time t. Assume that

  $(S_t;X_t)$ is jointly stationary

  I.e., S_t is in a stationary regime and X_t depends on the past, present and future state of the simulation in a way that is invariant by shift of time origin.
- Examples
  - S_t = current position of mobile, speed, and next waypoint
  - *Jointly stationary with* S_t: X_t = current speed at time t; X_t = time to be run until next waypoint
  - *Not jointly stationary with* S_t: X_t = time at which last waypoint occurred

Palm Expectation

- Consider some selected transitions of the simulation, occurring at times T_n.
  - Example: T_n = time of n^{th} trip end
- ***Definition*** : the Palm Expectation is

$$\mathbb{H}^t(X_t) = \mathbb{H}(X_t \mid \text{a selected transition occurred at time } t)$$

- By stationarity:

$$\mathbb{H}^t(X_t) = \mathbb{H}^0(X_0)$$

- Example:
  - T_n = time of n^{th} trip end, X_t = instant speed at time t
  - $\mathbb{H}^t(X_t) = \mathbb{H}^0(X_0)$ = average speed observed at a waypoint

Event versus Time Averages

- $H(X_t) = H(X_0)$ expresses the time average viewpoint.
- $H^t(X_t) = H^0(X_0)$ expresses the event average viewpoint.

- Example:
  - T_n = time of n^{th} trip end, X_t = instant speed at time t
  - $H^t(X_t) = H^0(X_0)$ = average speed observed at trip end
  - $H(X_t)=H(X_0)$ = average speed observed at an arbitrary point in time

Formal Definition

- In discrete time, we have an elementary conditional probability
  - $H^t(X_t) = H(X_t\ 1_{<\ n\ 5\ ]\ \text{such that } Tn=t})\ /\ S(<\ n\ 5\ ]$ such that $T_n=t)$

- In continuous time, the definition is a little more sophisticated
  - Similar to the definition of conditional density $f_X(x|Y=y)$ for continuous random variables with joint density – see the writeup [**L04]** for details
  - See [**BaccelliBremaud87**] for a formal treatment

- Palm probability is defined similarly
  - $S^t(\ X_t\ 5\ W) = H^t(1_{Xt\ 5\ W})$

Ergodic Interpretation

- Assume simulation is stationary + ergodic, i.e. sample path averages converge to expectations; then we can estimate time and event averages by:

$$\mathbb{E}(X_0) = \lim_{T \to +\infty} \frac{1}{T} \sum_{s=1}^{T} X_s$$

$$\mathbb{E}^0(X_0) = \lim_{N \to +\infty} \frac{1}{N} \sum_{n=1}^{N} X_{T_n}$$

- In terms of probabilities:

  - Stationary probability:
    $\mathbb{P}(X_t \in W) \approx$ fraction of time that X_t is in some set W
  - Palm probability:
    $\mathbb{P}^t(X_t \in W) \approx$ fraction of selected transitions at which X_t is in W

Two Palm Calculus Formulas

- Intensity of selected transitions: λ := expected number of transitions per time unit
- Intensity Formula:

$$\frac{1}{\lambda} = \mathbb{E}^0(T_1 - T_0) = \mathbb{E}^0(T_1)$$

where by convention T_0 ‰ $0 < T_1$

- Inversion Formula

$$\mathbb{E}(X_t) = \mathbb{E}(X_0) = \lambda \mathbb{E}^0 \left(\int_0^{T_1} X_s ds \right)$$

- The proofs are simple in discrete time – see [**L04**]

A Classical Example

- At bus stop in average λ buses per hour. Inspector measures time between all bus inter-departures. Inspector estimates $\mathbb{E}^0(T_1 - T_0) = \frac{1}{\lambda}$
- Joe arrives at time t and measures $X_t =$ (time until next bus $-$ time since last bus). Joe estimates $\mathbb{E}(X_0) = \mathbb{E}(T_1 - T_0)$
- Inversion formula:

$$\mathbb{E}(T_1 - T_0) = \lambda \mathbb{E}^0\left(\int_0^{T_1} X_t dt\right) = \lambda \mathbb{E}^0(T_1^2) = \frac{1}{\lambda} + \lambda \mathrm{var}^0(T_1 - T_0)$$

- Joe's estimate always larger than Inspector's (Feller's Paradox)

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Necessary Condition for Existence of a Stationary Regime

- Apply the intensity formula to T_n = trip end times

$$\frac{1}{\lambda} = \mathbb{E}^0(T_1 - T_0) = \mathbb{E}^0(T_1)$$

- Thus: if the random trip has a stationary regime it must be that the *mean trip duration* sampled at trip end times is *finite*

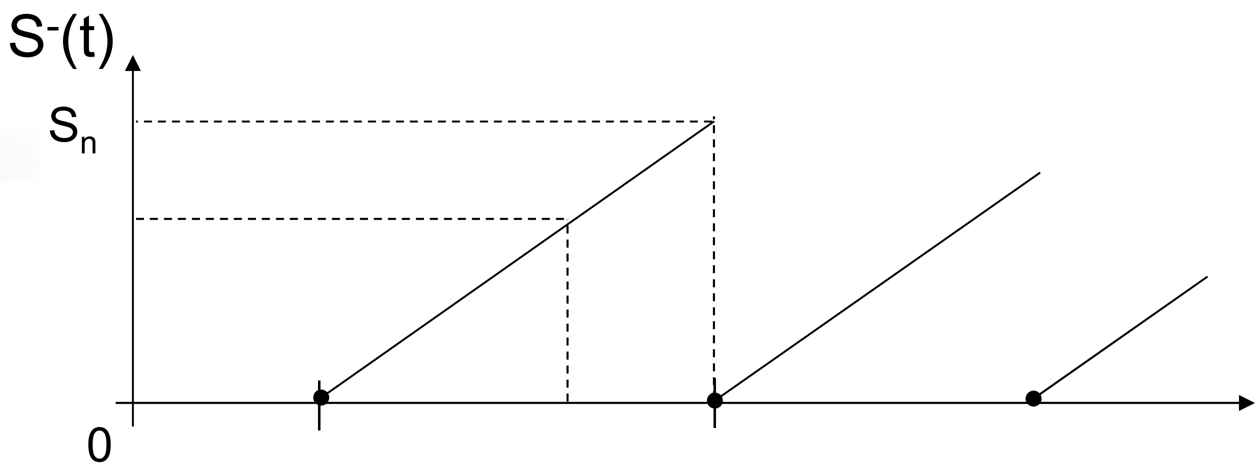
Formal Definition of Stability

- System state $\Phi(t) = (Y(t), S(t), S^{-}(t)), \quad t \geq 0$

(phase, path)

duration of current trip

time elapsed on current trip



- $\Phi(t)$ has
  - A unique time-stationary distribution π
  - The distribution of $\Phi(t)$ converges to π as t goes to infinity

Necessary and Sufficient Condition
Stability of random trip model [LV06]

- There exists a time-stationary distribution π for $\Phi(t)$ *if and only if mean trip duration is finite* (trip sampled at trip end times)

- Whenever π exists, it is unique

- Moreover, if mean trip duration is finite, from any initial state, the distribution of $\Phi(t)$ converges to π as t goes to infinity

Proof is based on Harris recurrence (see appendix)

Application to random waypoint

- Mean trip duration for a move
  = (mean trip distance) § mean of inverse of speed

- Mean trip duration for a pause
  = mean pause time

- Random waypoint is stable if both

  - mean of inverse of speed
  - mean pause time

  are finite

A Random waypoint model that has no time-stationary distribution !

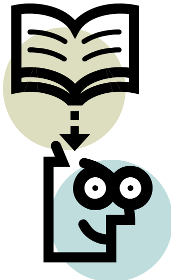
- Assume that at trip transitions, node speed is sampled uniformly on $[v_{min}, v_{max}]$

- Take $v_{min} = 0$ and $v_{max} > 0$ (common in practise)

- Mean trip duration = (mean trip distance) $\times \frac{1}{v_{max}} \int_0^{v_{max}} \frac{dv}{v} = +\infty$

- Mean trip duration is infinite !

- Speed decay: "considered harmful" [YLN03]



Stability of random trip model
» Summary «

- Random trip model is stable if mean trip duration is finite

- This ensures the model is stable
  - Unique time-stationary distribution, and
  - Convergence to this distribution from any initial state

- Didn’t hold for a random waypoint used by many

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Time-stationary distributions
›› Outline ‹‹

- Time-stationary distribution of node mobility state is the distribution of state in stationary regime, when it exists
- Should be used for fair comparison
- Can be obtained systematically by the Palm inversion formula

Example: Random Waypoint Distribution of Speed

- Assume stationary regime
- Apply inversion formula and obtain distribution of instantaneous speed V(t)

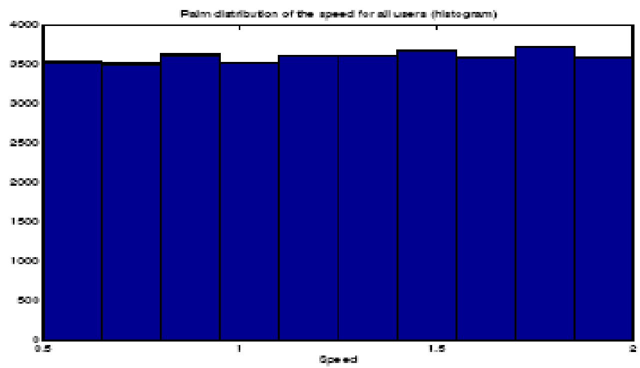
$$
\begin{aligned}
\mathbb{E}\left(\phi(V(t))\right) = \lambda \mathbb{E}^0 \left(\int_0^{T_1} \phi(V(t))\, dt \right) \\
&= \lambda \mathbb{E}^0 \left(\phi(V_0) T_1 \right) \\
&= \lambda \mathbb{E}^0 \left(\phi(V_0) \frac{\|M_1 - M_0\|}{V_0} \right) \\
&= \lambda \mathbb{E}^0 \left(\|M_1 - M_0\| \right) \mathbb{E}^0 \left(\frac{\phi(V_0)}{V_0} \right) \\
&= C \int_0^{v_{\max}} \frac{\phi(v)}{v} f^0_{V_0}(v)\, dv
\end{aligned}
$$

Inversion Formula Gives Relation between Speed Distributions at Waypoint and at Arbitrary Point in Time

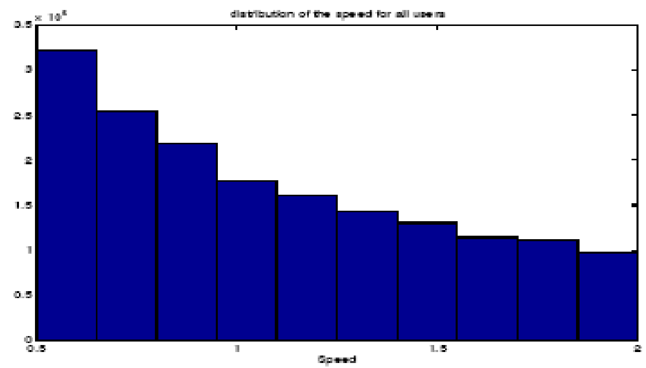
$$f_{V(t)}(v)dv = \frac{C}{v} f^0_{V_0}(v)dv$$

with: $f_{V(t)}(v)$ = stationary density of speed, $f^0_{V_0}(v)$ = Palm density of speed (i.e. uniform on $[v_{\min}, v_{\max}]$) and $C^{-1} = \mathbb{E}^0(\frac{1}{V_0})$

Event Average



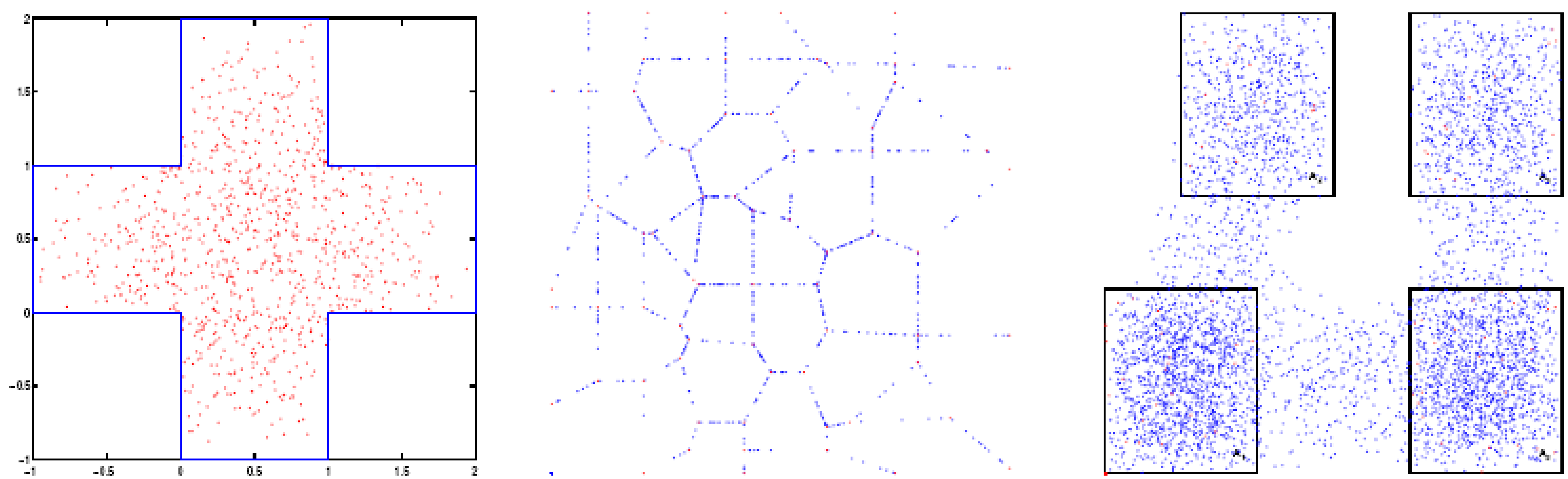
Time Average



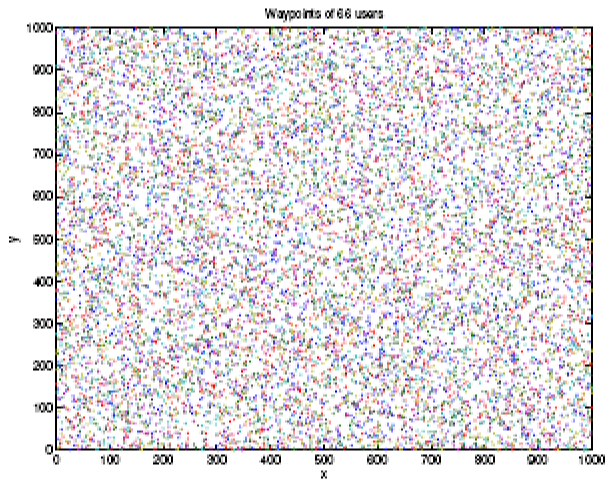
Sampling bias is also for location

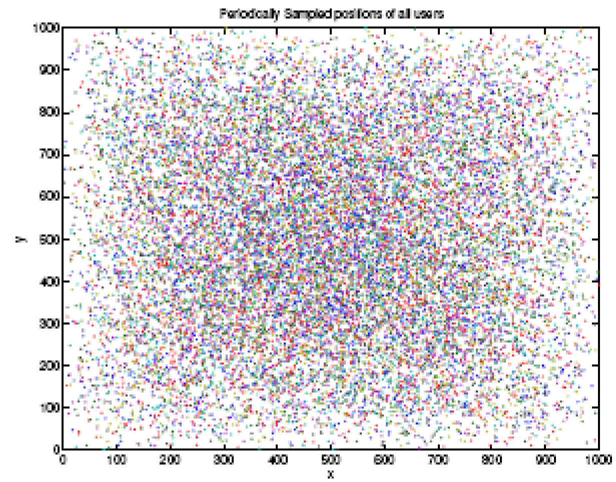
- Stationary distributions at arbitrary times and at trip end points are not necessarily the same
  - Time-average vs event-average

- Ex. samples of node position for random waypoint
  - Trip endpoints are uniformly distributed, time stationary distribution of mobile location is *not*



Approximate formulae for location



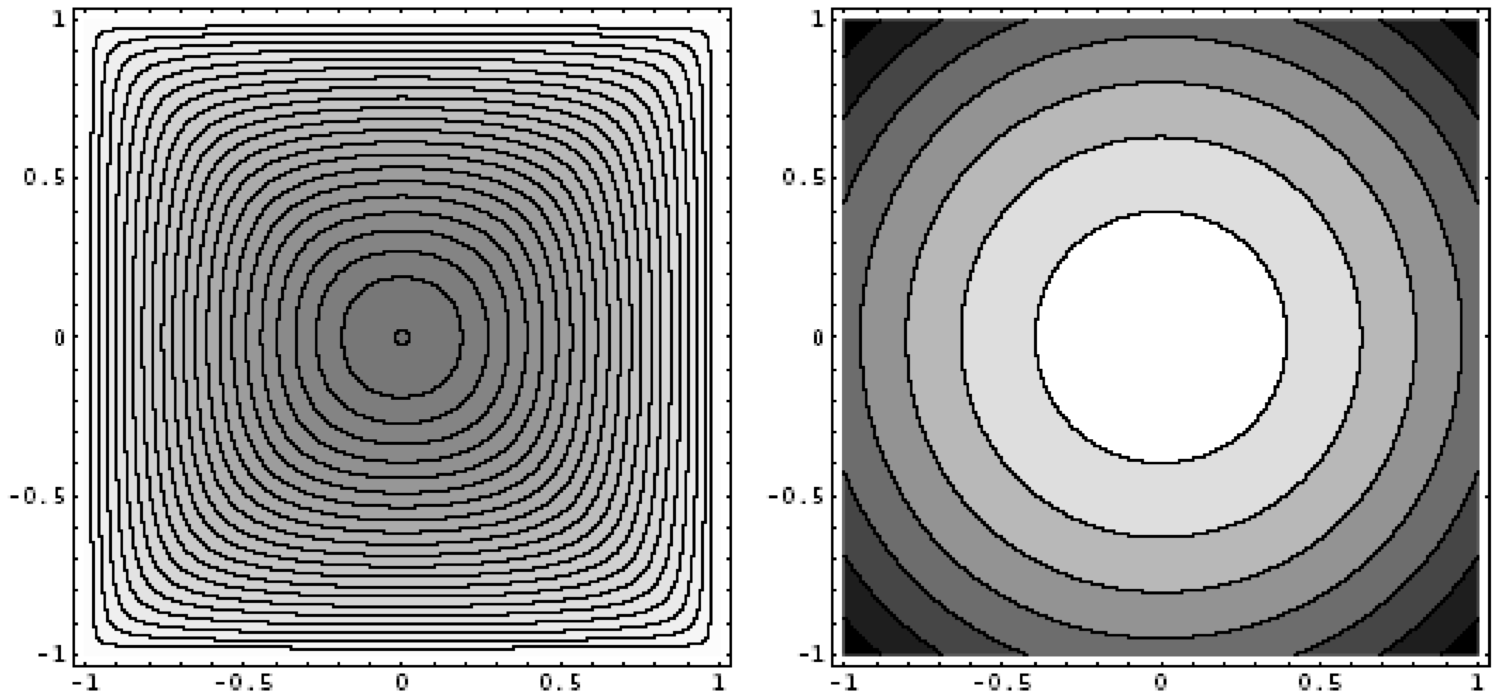


- Conventional approaches find that closed form expression for density is too difficult [Bettstetter04]
- Approximation of density in area [0; a] [0; a] [Bettstetter04]:

$$f_{X,Y}(x,y) \approx \frac{36}{a^2} x(x-a)y(y-a)$$

Inversion formula also gives stationary distribution of random waypoint location in closed form [L04]

Contour plots of density of stationary distribution



(a) Node Location

(b) Next Waypoint

Closed forms

$$\begin{cases} f_{M(t)}(x,y) = f_{M(t)}(|x|,|y|) \\ \text{if } |x| < |y| \text{ then } f_{M(t)}(x,y) = f_{M(t)}(|y|,|x|) \\ \text{if } 0 \leq y \leq x \text{ then } f_{M(t)}(x,y) = \frac{15}{32\left(\sqrt{2}+2+5\ln(1+\sqrt{2})\right)} F(x,y) \end{cases}$$

with $F(x,y) =$

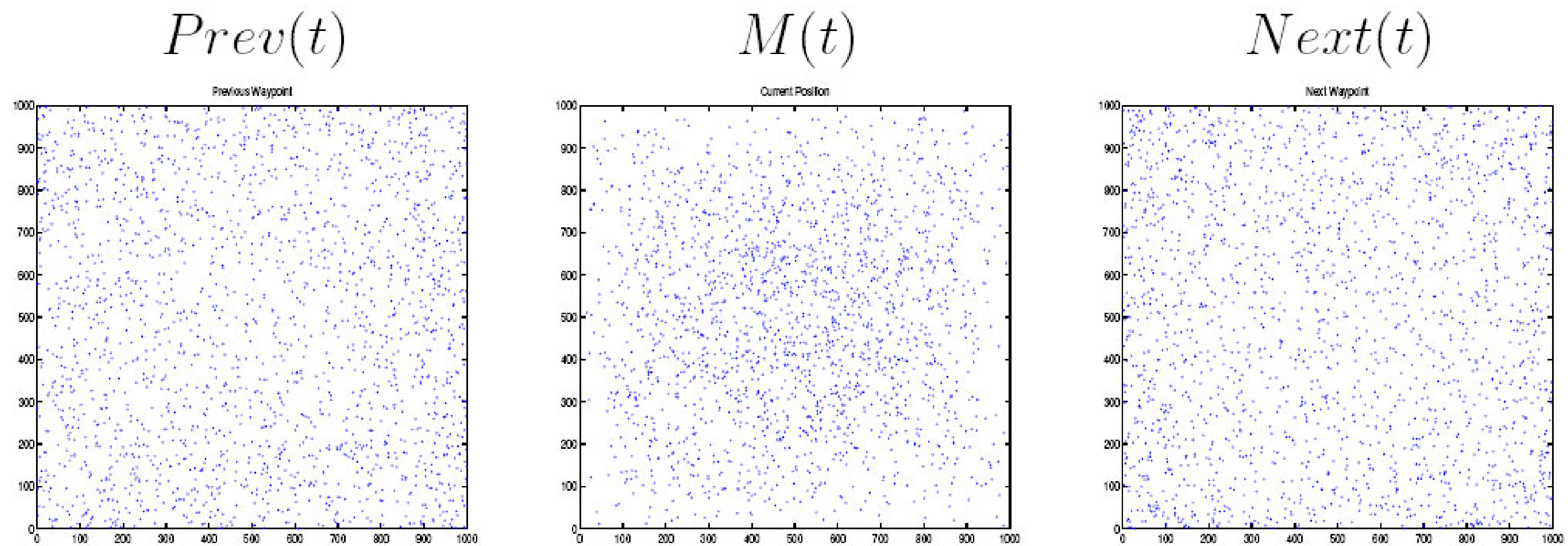
$$\begin{array}{rrlrrl}
 & (1-x)(2+x)(1-y) & \sqrt{1+\frac{(1-y)^2}{(1+x)^2}} & + & (1-x)(1-y)(2+y) & \sqrt{1+\frac{(1-x)^2}{(1+y)^2}} \\
+ & (1-x)(2+x)(1+y) & \sqrt{1+\frac{(1+y)^2}{(1+x)^2}} & + & (1-x)(1+y)(2-y) & \sqrt{1+\frac{(1-x)^2}{(1-y)^2}} \\
- & \frac{(1-x)^2(1-y)^2}{1+x} & \sqrt{1+\frac{(1+x)^2}{(1-y)^2}} & - & \frac{(1-x)^2(1-y)^2}{1+y} & \sqrt{1+\frac{(1+y)^2}{(1-x)^2}} \\
- & \frac{(1-x)^2(1+y)^2}{1+x} & \sqrt{1+\frac{(1+x)^2}{(1+y)^2}} & - & \frac{(1-x)^2(1+y)^2}{1-y} & \sqrt{1+\frac{(1-y)^2}{(1-x)^2}} \\
+ & (1-x)\left[1+x-(1-y)^2\right] & \sinh^{-1}\left(\frac{1-y}{1+x}\right) & + & (1-y)\left[1+y-(1-x)^2\right] & \sinh^{-1}\left(\frac{1-x}{1+y}\right) \\
+ & (1-x)\left[1+x-(1+y)^2\right] & \sinh^{-1}\left(\frac{1+y}{1+x}\right) & + & (1+y)\left[1-y-(1-x)^2\right] & \sinh^{-1}\left(\frac{1-x}{1-y}\right) \\
+ & (1-x)^2(1-y) & \sinh^{-1}\left(\frac{1+x}{1-y}\right) & + & (1-x)(1-y)^2 & \sinh^{-1}\left(\frac{1+y}{1-x}\right) \\
+ & (1-x)^2(1+y) & \sinh^{-1}\left(\frac{1+x}{1+y}\right) & + & (1-x)(1+y)^2 & \sinh^{-1}\left(\frac{1-y}{1-x}\right)
\end{array} \tag{14}$$

where $\sinh^{-1}(t) = \ln\left(t+\sqrt{1+t^2}\right)$ *(inverse hyperbolic sine).*

Previous and Next Trip Endpoints

Ex: random waypoint

- Let $M(t)$: position at time t
- Let $Prev(t), Next(t)$: previous and next waypoints



A No. But $Prev(t)$ and $Next(t)$ have same (non uniform) distribution.

Inversion formula gives simple expression for time-sationary distribution of complete state

(restricted) random waypoint on arbitrary area:

- Conditional on phase is (i, j, r, move) ([NavidiCamp04] for rwp)

- Node speed at time t is independent of path and location with density

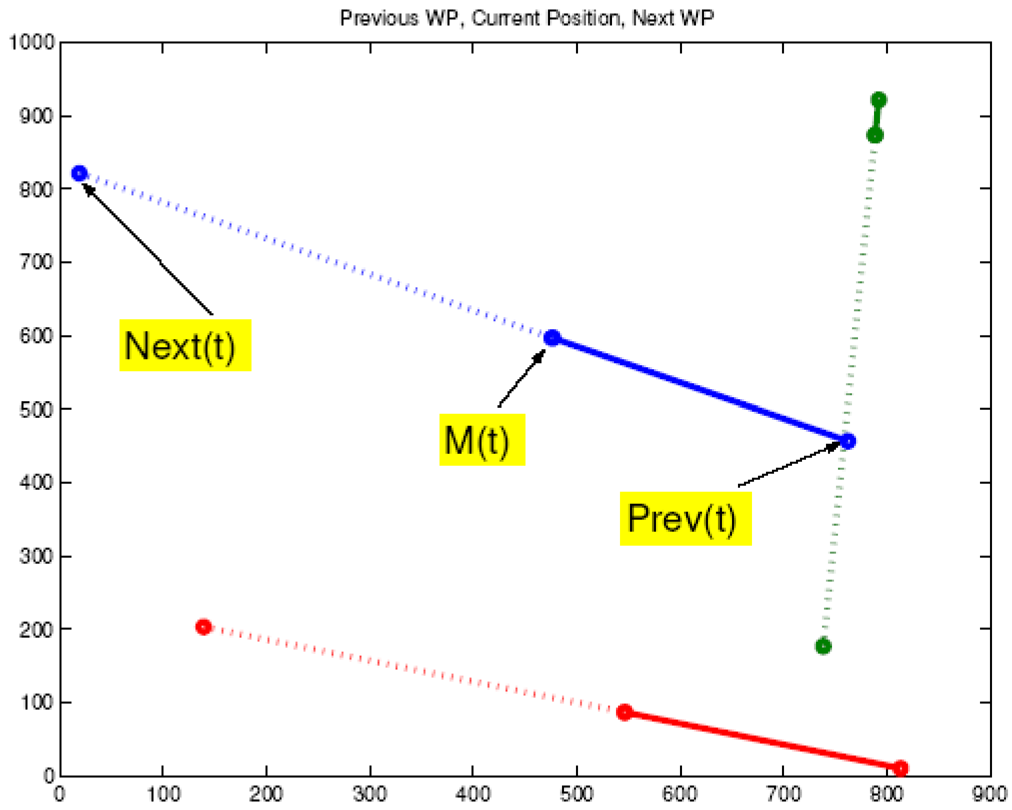
$$f_V(v) = \text{const}\, \frac{1}{v} f_0^V(v)$$

- Path endpoints at time t, (P(t)(0),P(t)(1)) = (m_0,m_1) have a joint density:

$$= K_{ijr}\; d(m_0, m_1), \quad \text{for } (\mathrm{m}_0, \mathrm{m}_1) \in \mathrm{A}_\mathrm{i} \times A_j$$

- Conditional on (P(t)(0),P(t)(1))=(x,y), distribution of node position X(t) is uniform on the segment [x,y]

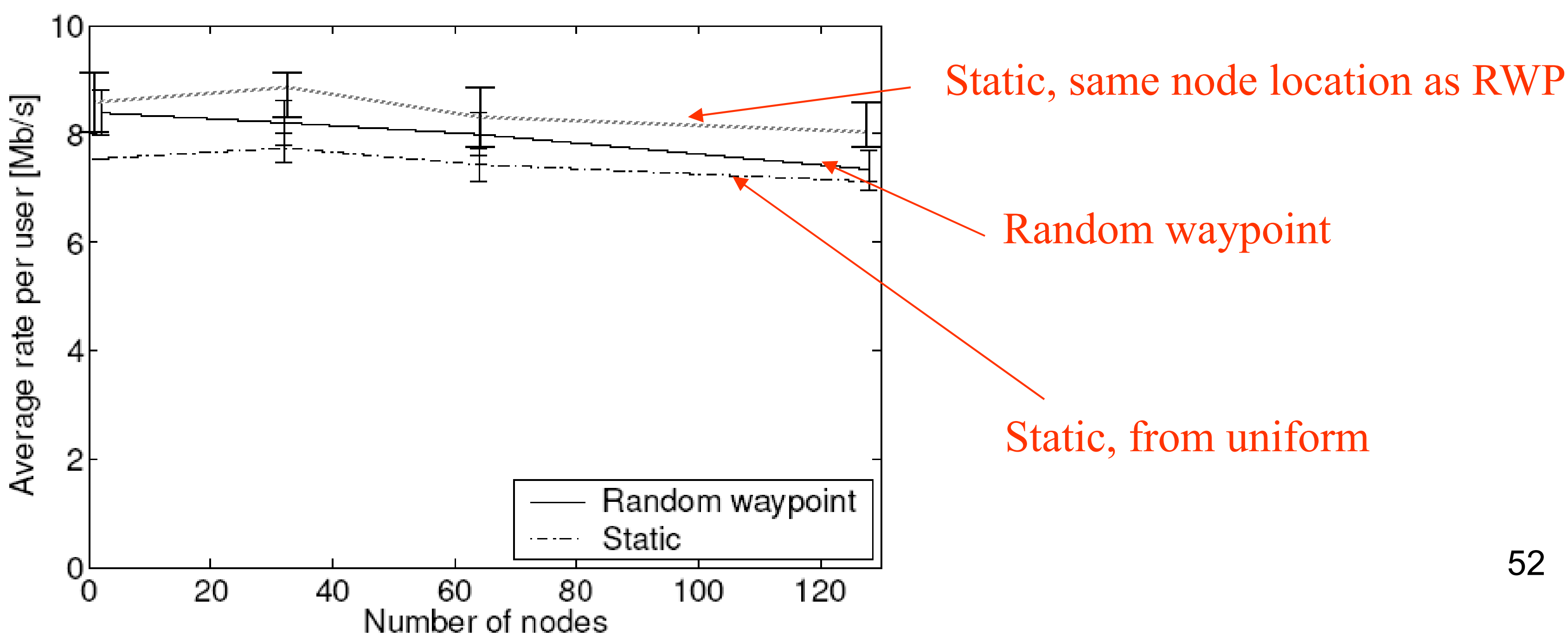
Stationary Distribution of Location



- Joint distribution of (Next, Prev) has density proportional to distance
- Given (Next, Prev), M is uniformly distributed

A Fair Comparison

- If there is a stationary regime, we can compare different mobility patterns provided that
  1. They are in the stationary regime
  2. They have the same stationary distributions of locations
- Example: we revisit the comparison by sampling the static case from the stationary regime of the random waypoint



Representation of time-stationary distribution (any random trip model)

- Phase:

$$P(I(t)=i)=\frac{\pi^0(i)\tau_i}{\sum_j \pi^0(j)\tau_j}$$

where $\tau_i = E^0(S_0 \mid I_0 = i)$, i.e. mean trip duration given that phase is i

- Path and duration, given the phase:

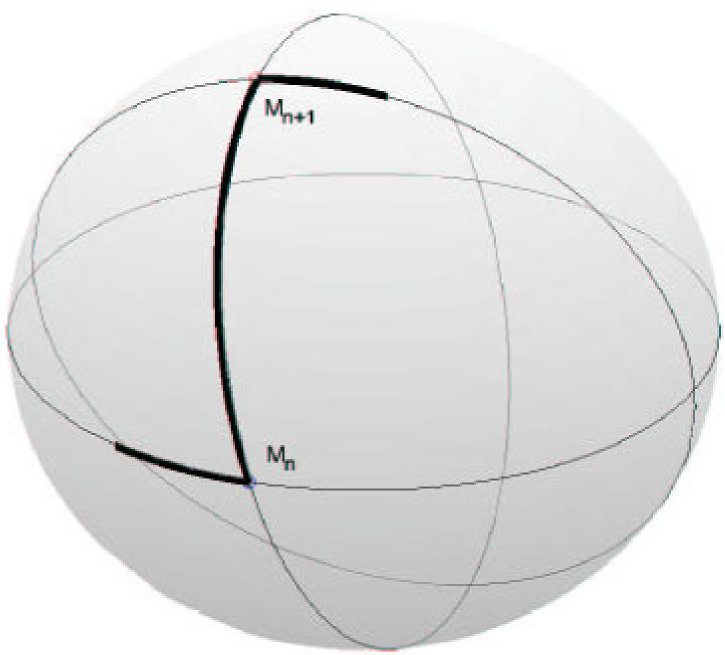
$$dP(P(t)=p, S(t)=s \mid I(t)=i)=\frac{s}{\tau_i} dP^0(P_0 = p, S_0 = s \mid I_0 = i)$$

- Time elapsed on the current trip: $S^-(t) = S(t)U(t)$, where U(t) is uniform on [0,1]

Models with Uniform Location

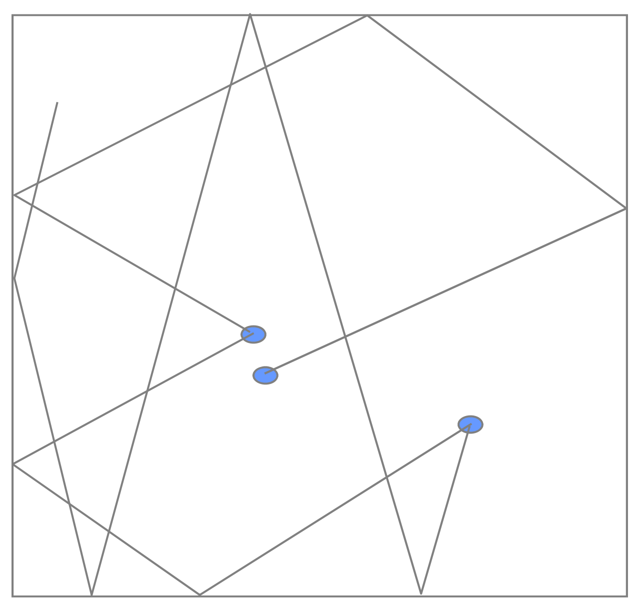
Random waypoint on sphere

- Time stationary distribution of location is uniform
- But trip endpoints are not



Billiards, Random waypoint on torus

- Time stationary distribution of location is uniform
- So are trip endpoints



Time-stationary distributions
» Summary «

- Palm inversion yields representation of time-stationary distribution for *any* random trip model

- Representation can be used to derive closed form for location alone (*painful* and *useless*)

- Representation can be used to derive closed form for complete state (*easy* and *useful*) – see next section

Outline

- Simulation Issues with mobility models
- Random trip basic constructs
- Palm calculus instant primer
- Stability condition for random trip
- Time-stationary distributions
- Perfect simulation

Appendix

- FAQ
- Harris recurrence

Perfect simulation
›› Outline ‹‹

- Perfect simulation
  - Sample initial state from time-stationary distribution
  - Then state is a time-stationary realization at any time

  - If you know how to do perfect simulation, there is *no transient*

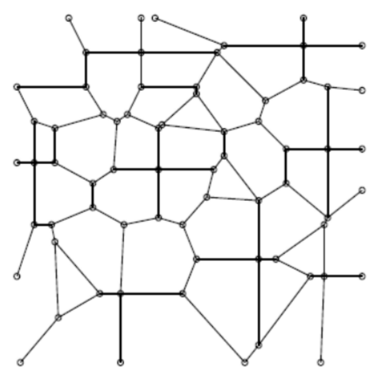
Perfect simulation is highly desirable

- If model is stable and initial state is drawn from distribution other than time-stationary distribution
  - The distribution of node state converges to the time-stationary distribution

- Naïve: so, let’s simply truncate an initial simulation duration

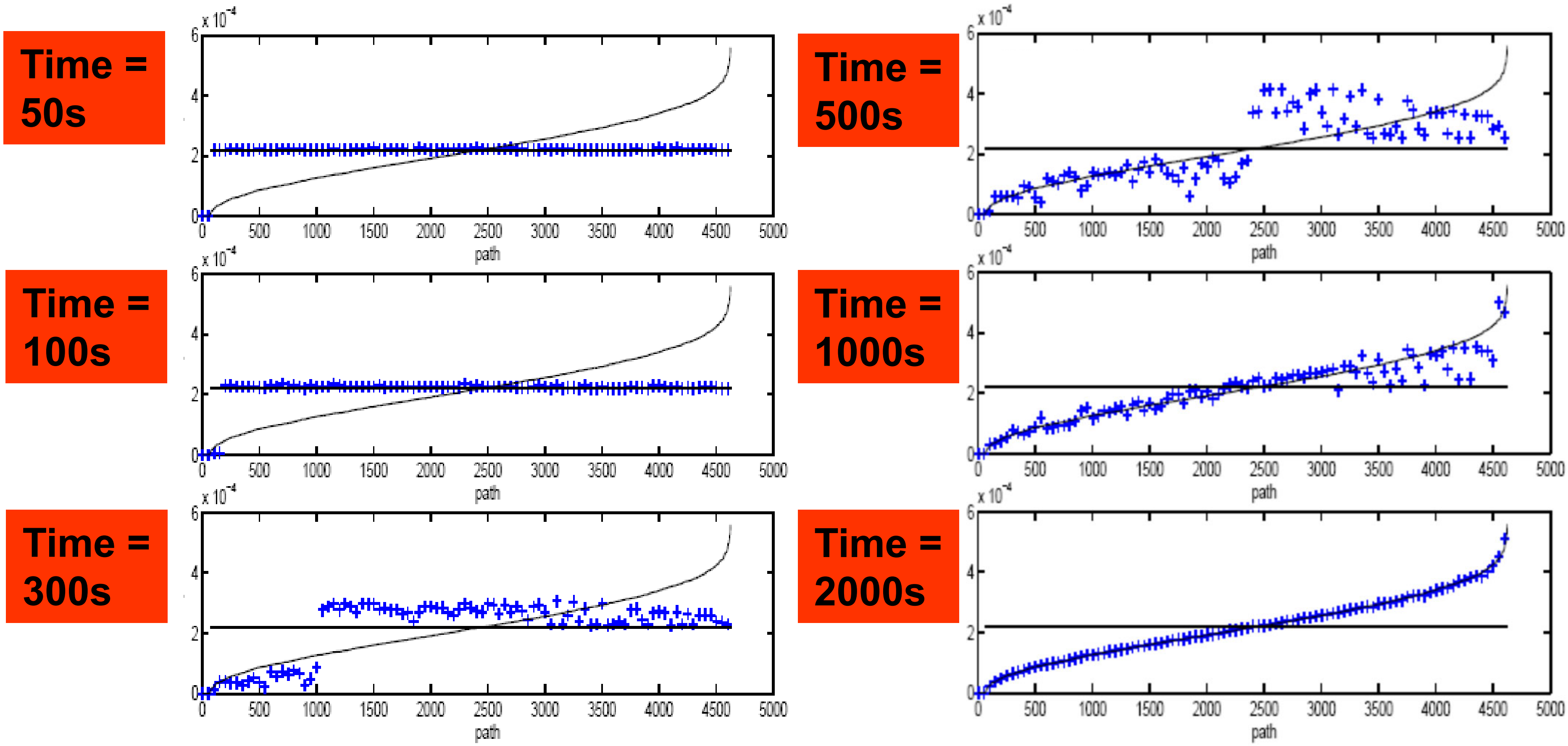
- The problem is that initial transience can last very long

Example [space graph]:
node speed = 1.25 m/s
bounding area = 1km x 1km



Perfect simulation is highly desirable (2)

- Distribution of path:



Perfect Simulation Based on the Representation of Stationary Distributions

Time-stationary distribution fo (restricted) random waypoint

- Conditional on phase is (i, j, r, move)

- Node speed at time t is independent of path and locat with density

$$f_V(v) = \text{const}\, \frac{1}{v} f_0^V(v)$$

- Path endpoints at time t, (P(t)(0),P(t)(1)) = (m_0,m_1) I a joint density:

$$= K_{ijr}\, d(m_0, m_1), \quad \text{for } (m_0, m_1) \in A_i \times A_j$$

- Conditional on (P(t)(0),P(t)(1))=(x,y), distribution of position X(t) is uniform on the segment [x,y]

- A similar formula exists for the general random trip m (next slide)

- **Question**: how to sample (M_0, M_1) when we know joint pdf is

  $$f_{(M0,\ M1)}(m_0, m_{\mathbf{1}}) = K\, g\,(m_0, m_{\mathbf{1}})$$

  g() easy to compute

- **Answer**: rejection sampling

  do
      sample $m_0, m_1 \sim$ unif(A)
      sample $V \sim$ unif([0, Δ]))
  until $V < g(m_0, m_1)$

  with Δ upper bound on g

Perfect sampling algorithm for random waypoint

Input: A, Δ

Output: X_0, X, X_1

1. Do

   sample X_0, X_1, iid, $\sim$ Unif(A)

   sample V $\sim$ Unif$[0, \Delta]$

   until $V < ||X_1 - X_0||$

2. Draw U $\sim$ Unif[0,1]

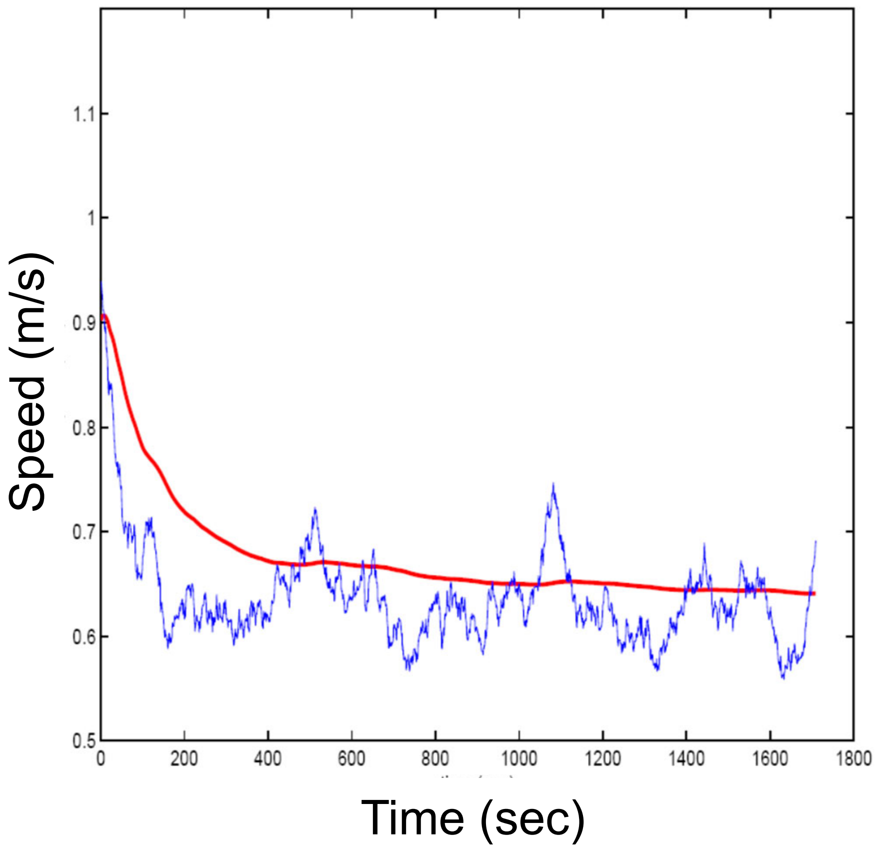
3. $X = (1-U)\, X_0 + U\, X_1$

Input: A = domain, Δ = upper bound on the diameter of A

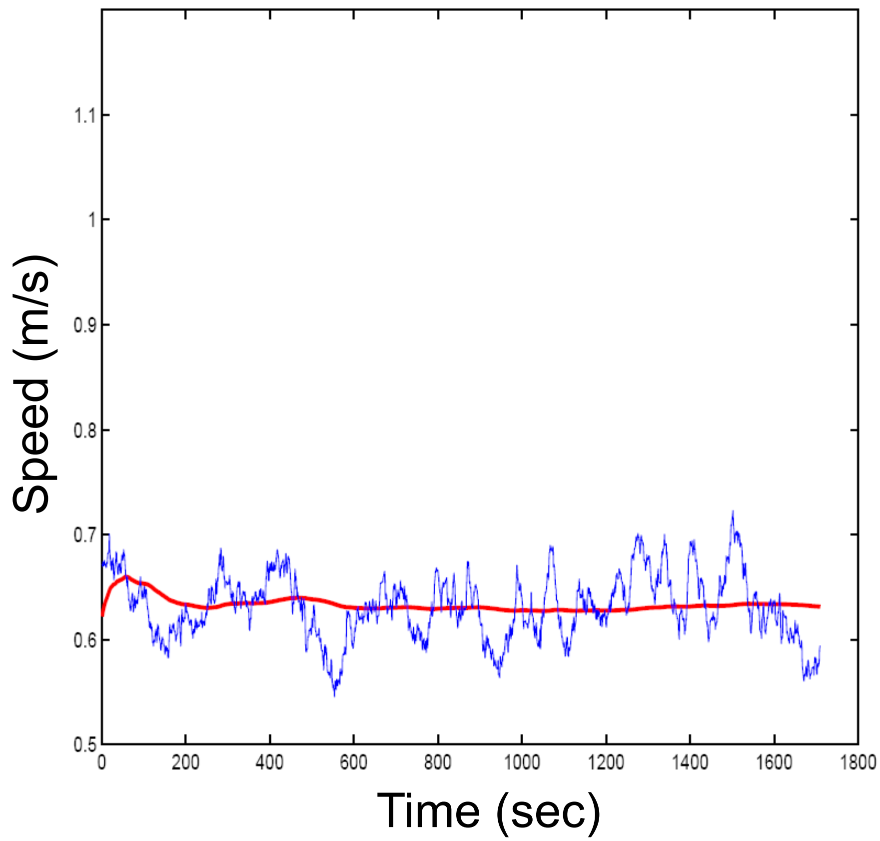
Similar algo exists for *any* random trip model

Example: random waypoint
No speed decay

- Standard simulation

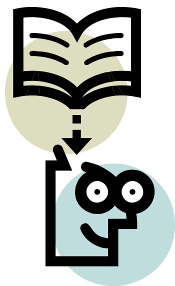


- Perfect simulation



Perfect simulation software

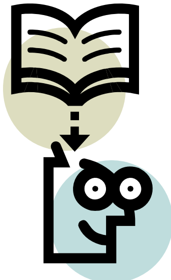
- Developed by Santashil PalChaudhuri
  - see the random trip web page
- Scripts to use as front-end to ns-2
  - Output is ns-2 compatible format to use as input to ns-2
- Supported models:
  - Random waypoint on general connected domain
  - Restricted random waypoint
  - Random walk with wrapping
  - Billiards



Perfect simulation
» Summary «

- Random trip model can be perfectly simulated
  - Node mobility state is a time-stationary realization throughout a simulation

- Perfect simulation by rejection sampling
  - It alleviates knowing geometric constants
  - Bound on the trip length is sufficient



Concluding remarks

- Random trip model covers a broad set of models of *independent* node movements
  - All presented in the catalogue of this slide deck
- Defined by a set of stability conditions
- Time-stationary distributions specified by Palm inversion
- Sampling algorithm for perfect simulation
  - No initial transience
  - Not necessary to know geometric constants

Future work

- Realistic mobility models ?
- Real-life invariants of node mobility ?
  - Human-carried devices, vehicles, …
- What extent of modelling detail is enough ?
- Scalable simulations ?
- Algorithmic implications ?
- Scalable simulations ?
- Statistically dependent node movements
  - Application scenarios, models ?

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Outline

- Simulation Issues with mobility models
- Random trip basic constructs
- Palm calculus instant primer
- Stability condition for random trip
- Time-stationary distributions
- Perfect simulation

Appendix

- FAQ
- Harris recurrence

Appendix 1: Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?
- Does model accommodate heavy-tailed trip durations ?
- Can model produce a given time-stationary distribution of node position ?
- What are mobility data resources ?

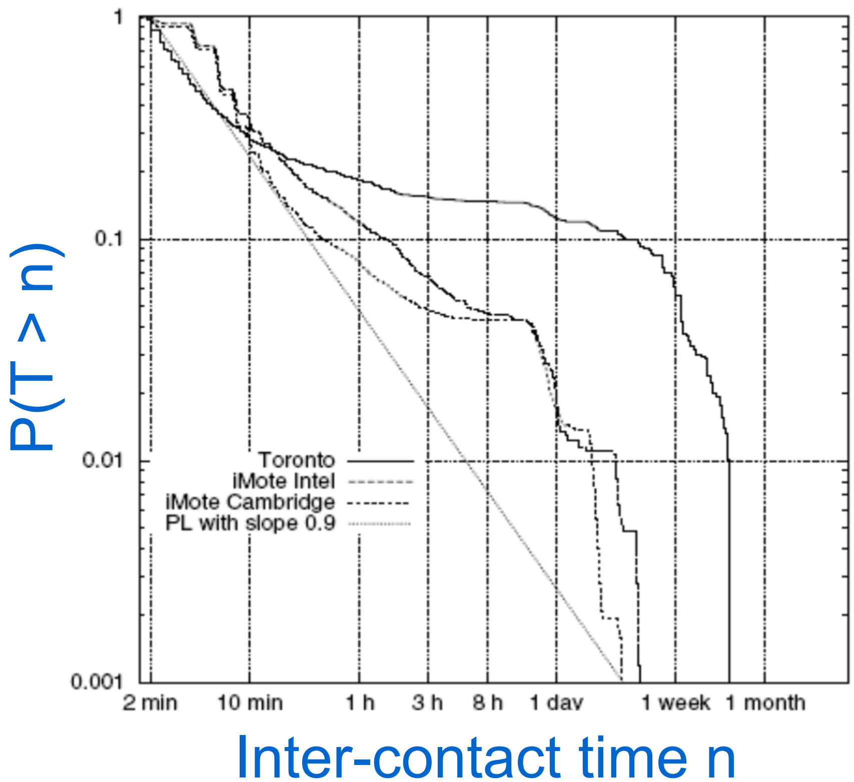
Frequently Asked Questions

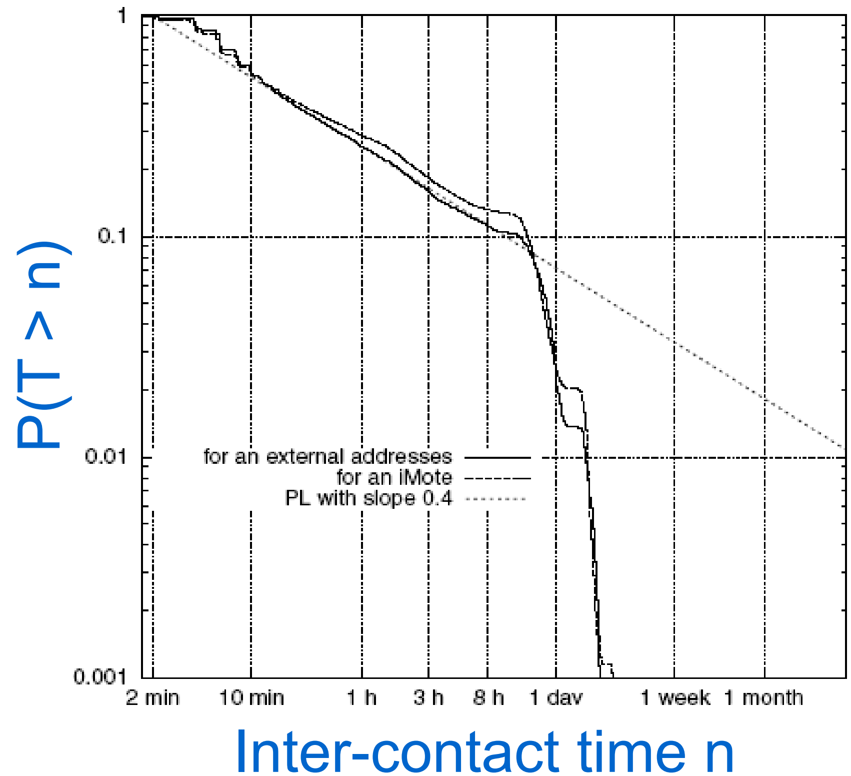
Does model accommodate power-law inter-contact times ?

- Does model accommodate heavy-tailed trip durations ?
- Can model produce a given time-stationary distribution of node position ?
- What are mobility data resources ?

Power-law evidence

- Chaintreau et al 2006 [CHC+06]: distribution of inter-contact times of human carried devices (iMote/PDA) is well approximated by a power law





- Source [CHC+06] with permission from authors

Power-law inter-contact times (cont’d)

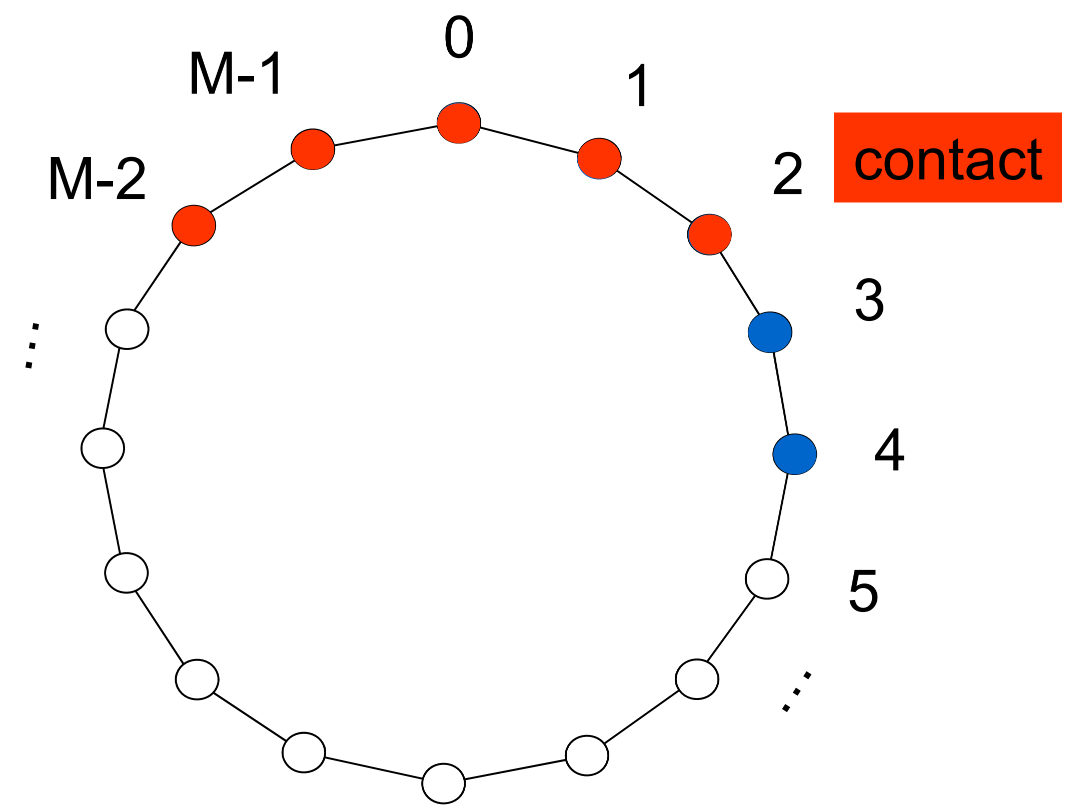
- Implications on packet-forwarding delay ([CHC+06])

?	Can random trip model accommodate power-law node inter-contacts ?

  - Yes ! (see next example)

Example: random walk on torus

- Discrete-time, discrete-space of M sites



- T = inter-contact time, $E(T) = M$

Example: random walk on torus (2)

- Let first $M \to \infty$ (infinite lattice)

  $P(T > n) \sim const / n^{1/2}$, large n

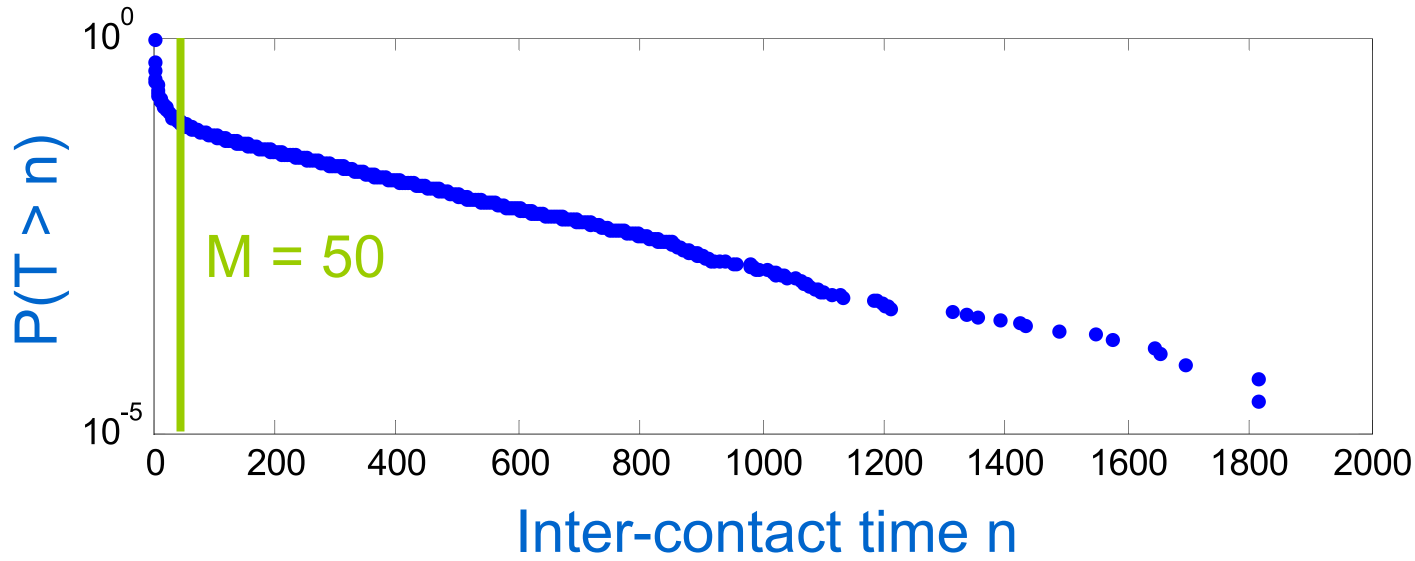
  power-law

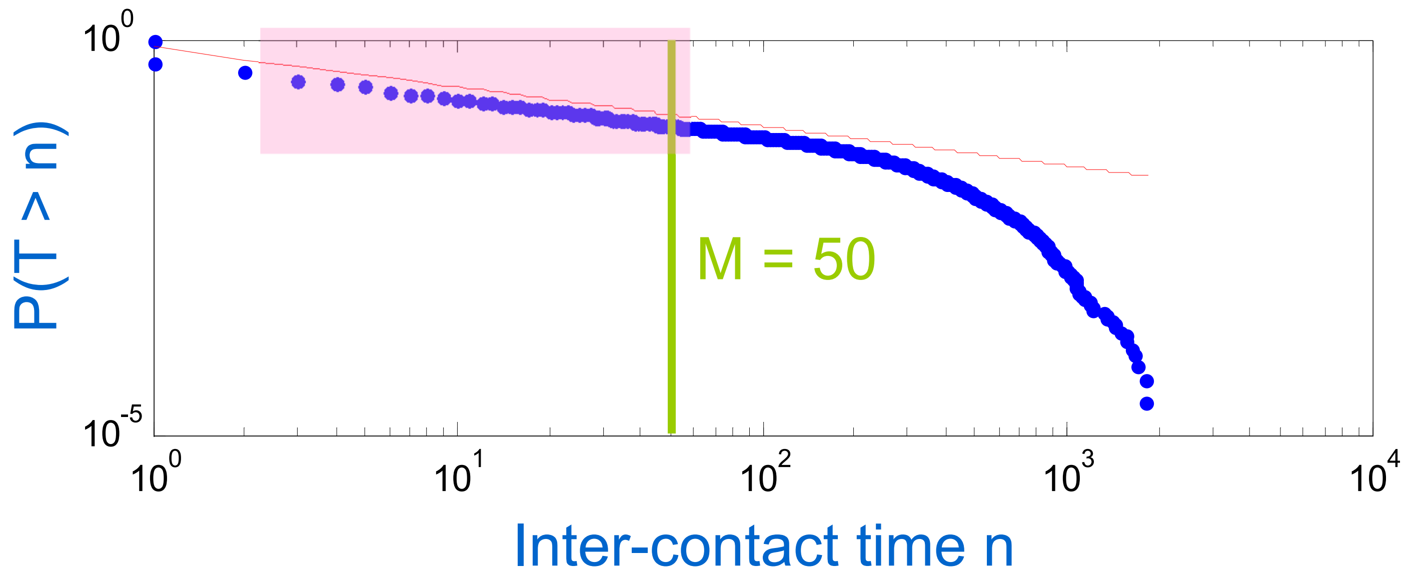
  – Holds for any aperiodic recurrent random walk with finite variance on infinite 1dim lattice, Spitzer [S64]

- If M is fixed, tail is exponentially bounded

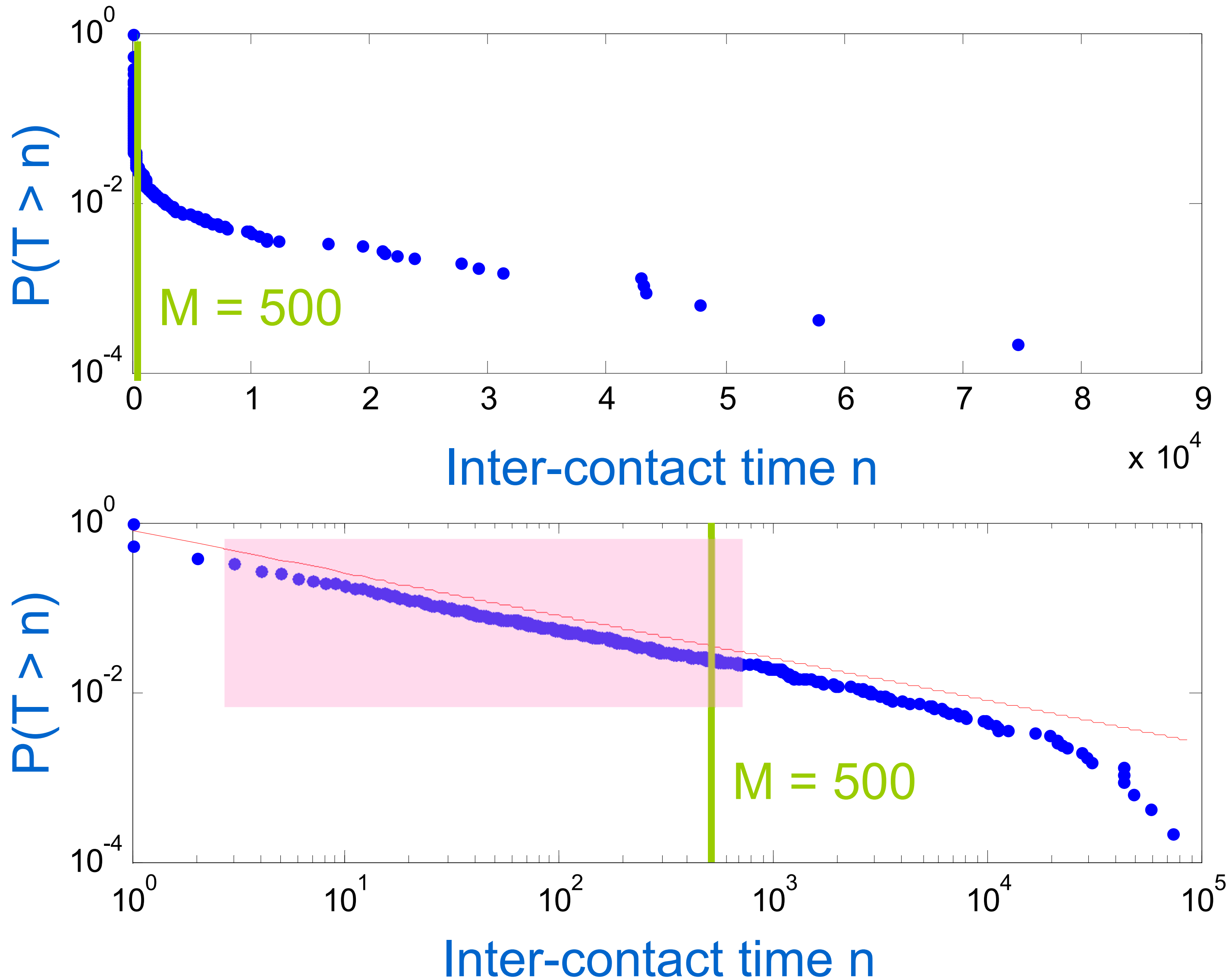
- If n and M scale simultaneously ? (see next)

Example: random walk on torus (3) M = 50

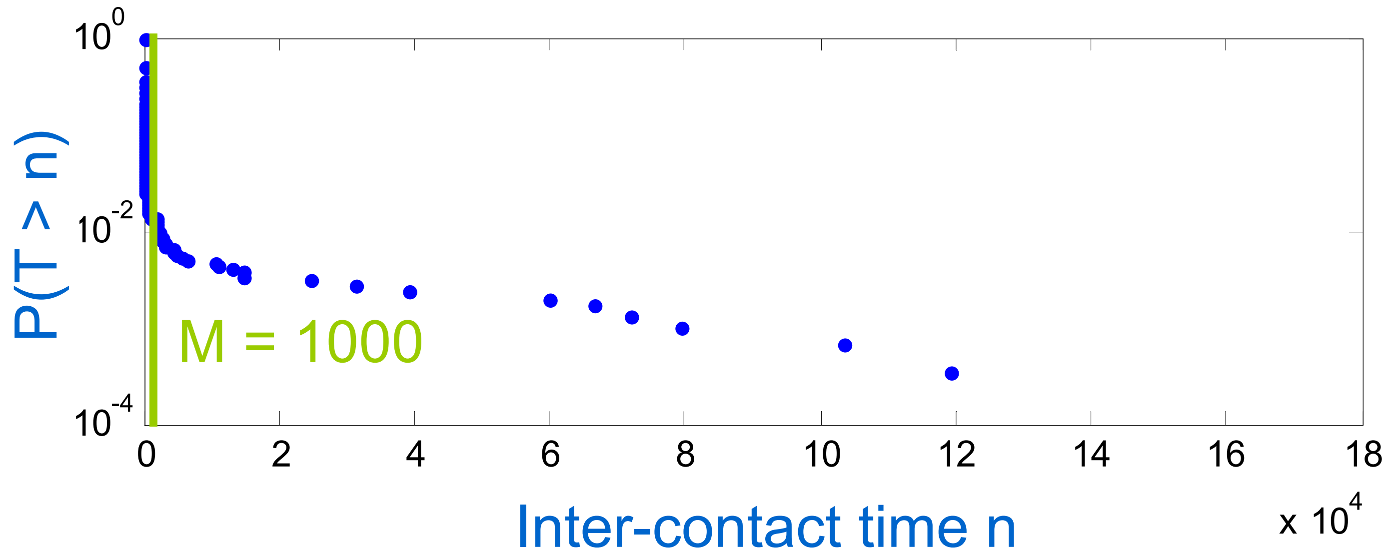


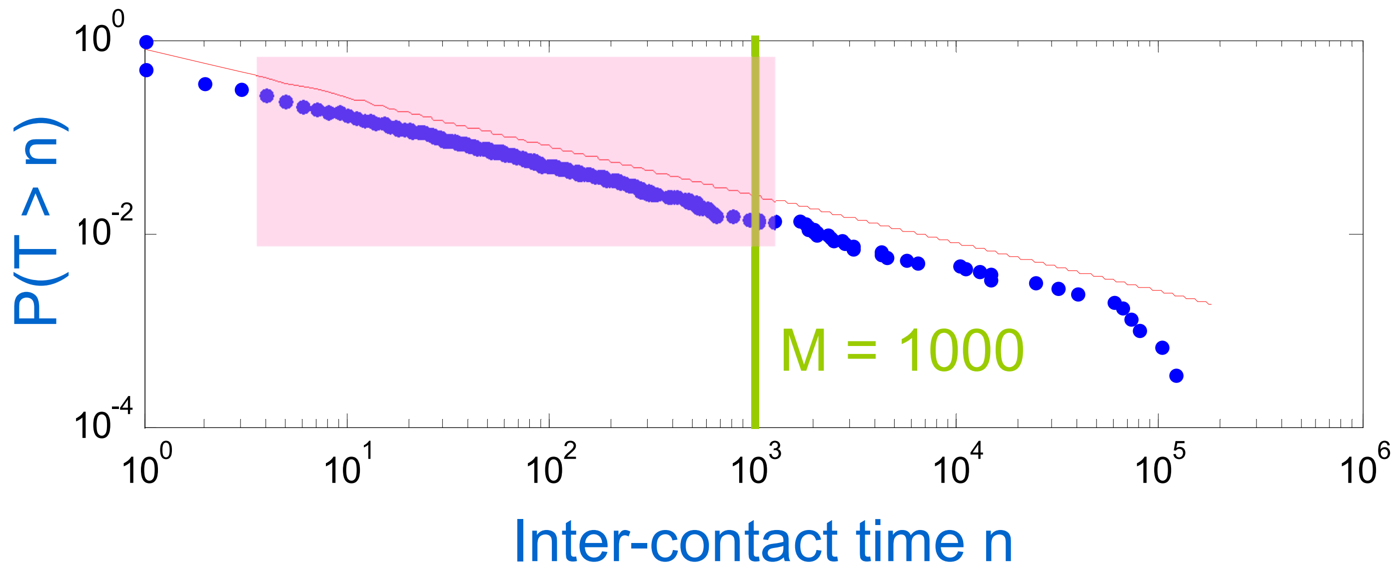


Example: random walk on torus (4) M = 500



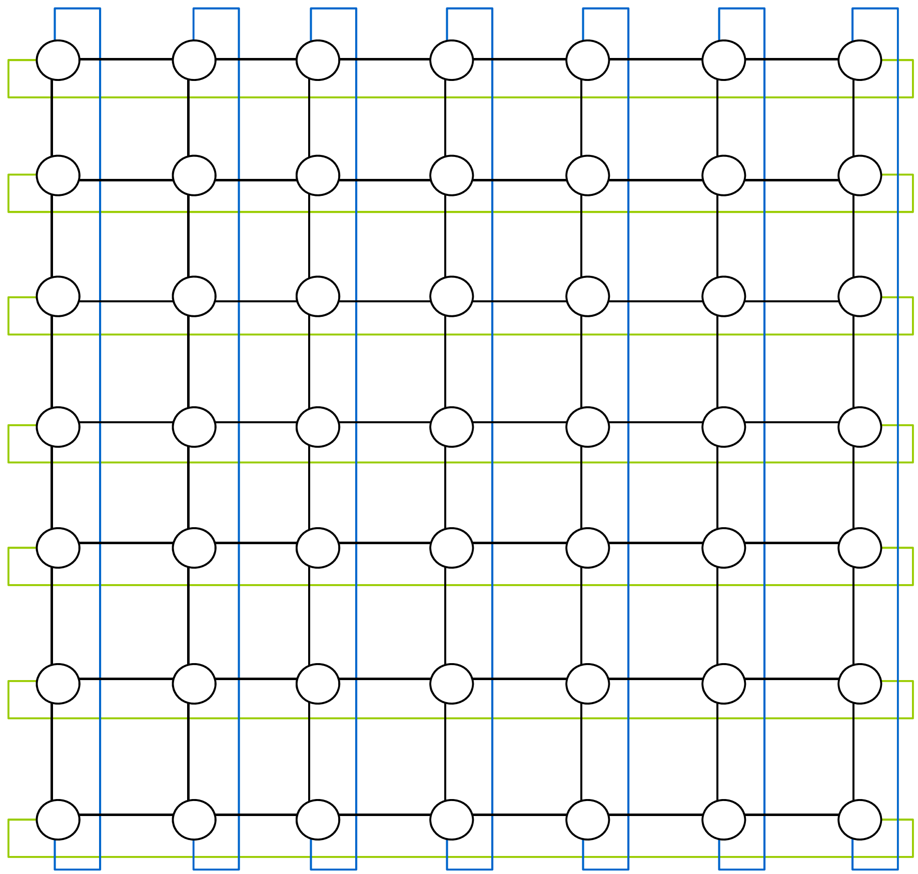
Example: random walk on torus (4)
M = 1000





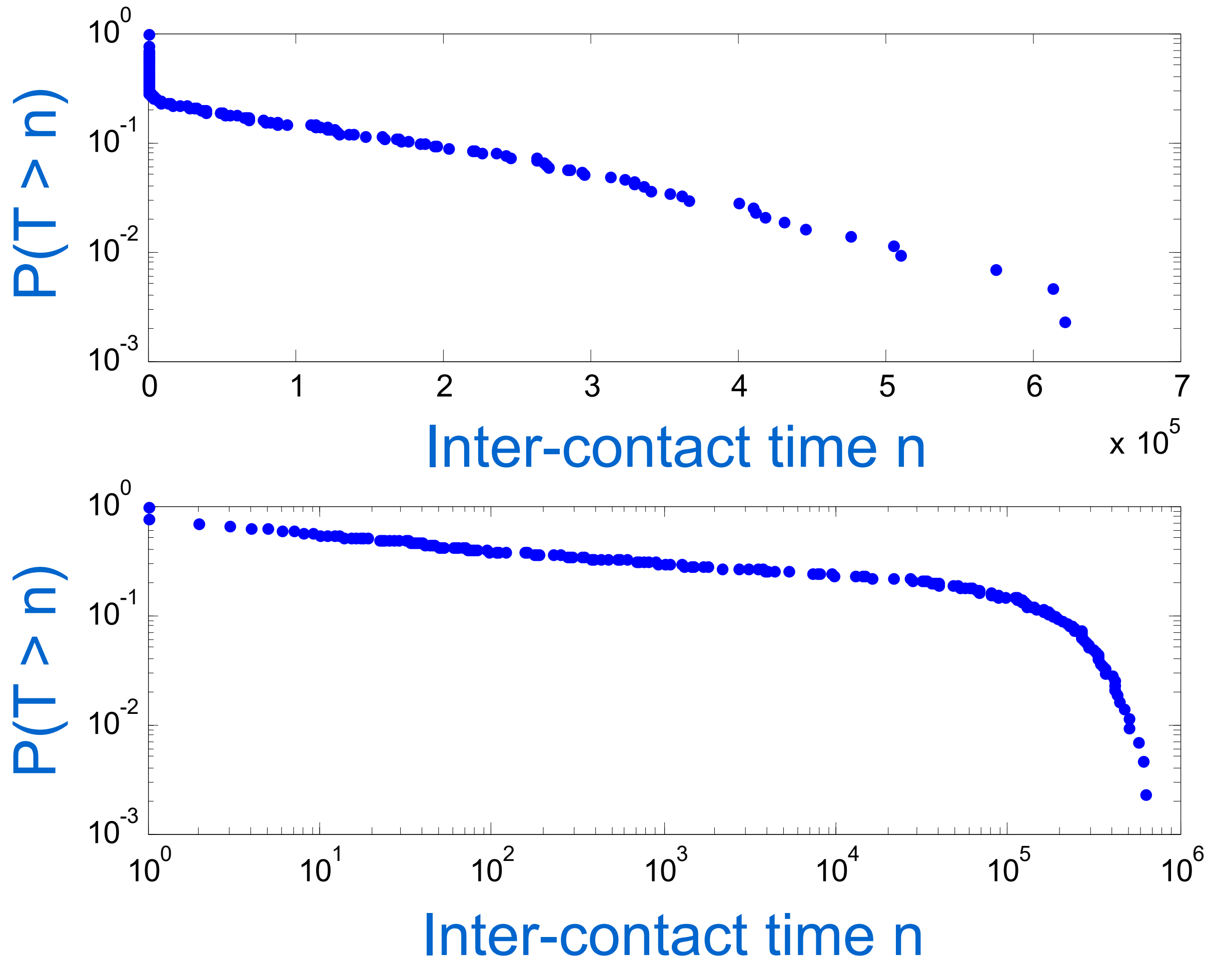
What if random walk is on a 2dim torus ?

- Manhattan grid
- Ex [M87], [SMS06]



What if random walk is on a 2dim torus ? (2)

- Finite torus: 500 x 500 (20M walk steps)



Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?

Does model accommodate heavy-tailed trip durations ?

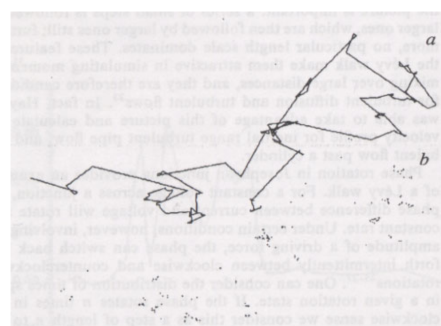
- Can model produce a given time-stationary distribution of node position ?

- What are mobility data resources ?

Heavy-tailed trip times

?	Can trip duration be heavy-tailed ? – Yes.

- Common in nature
  - Albatross search, spider monkeys [KS05], jackals [ARMA02]
  - Model: random walk with heavy-tailed trip distance (Levy flights)



Levy flight (source [FZK93])

Heavy-tailed trip times (2)

- Ex 1: random walk on torus or billiards
  - Take a heavy-tailed distribution for trip duration with finite mean
  - Ex. Pareto: $P^0(S_n > s) = (b/s)^a$, $b > 0, 1 < a < 2$
- Ex 2: Random waypoint
  - Take $f_V^0(v) = K \; v^{1/2} \; 1(0 \leq v \leq vmax)$
  - $E^0(S_n) < \infty$, $E^0(S_n^2) = \infty$

Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?
- Does model accommodate heavy-tailed trip durations ?

Can model produce a given time-stationary distribution of node position ?

- What are mobility data resources ?

Given time-stationary distribution of node position

- Given is a random trip model with time-stationary density of node position $a_X(x)$

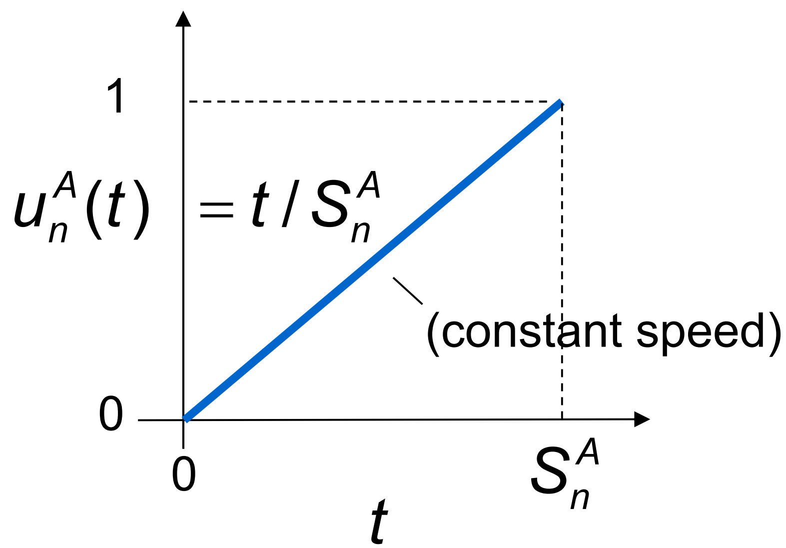
?	Can one configure the model so that time-stationary density of node position is a given $b_X(x)$?

  - Yes. Twist speed as described next

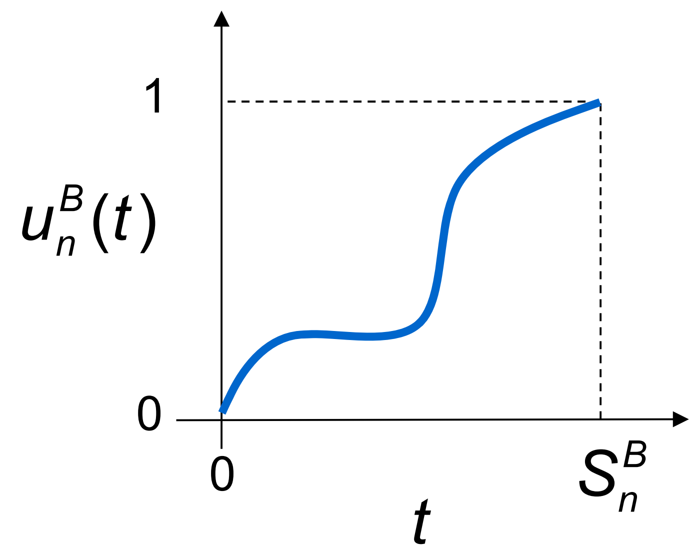
  Remarks:
  - Speed twisting applies to random trip model, in general
  - See [GL06], for random direction model

Speed twist

A: original model



B: twisted model



t = time elapsed on trip

u_n^A, u_n^B = fraction of traversed trip length

- Twist function $u_n^B(t)$?

Speed twist (2)

- Palm inversion formula: the twist function is given by differential equation:

$$\frac{d}{dt}u_n^B = \frac{1}{S_n^A} w\left(P_n\left(u_n^B\right)\right), \quad 0 \leq t \leq S_n^B$$

with boundary values $u_n(0) = 0$, $u_n(S_n^B) = 1$

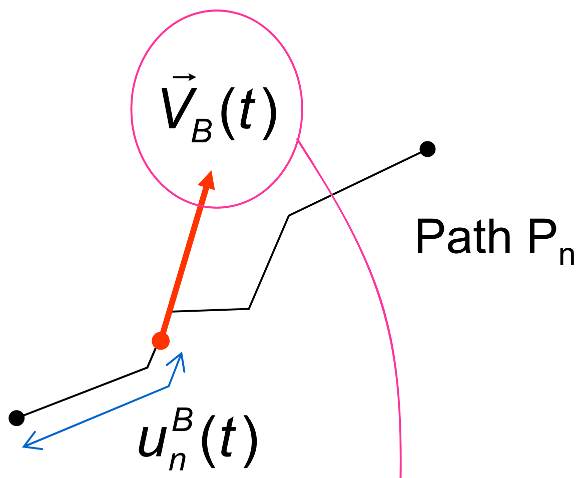
and $w(x) := a_X(x) / b_X(x)$

- Trip duration may change but its mean remains same:

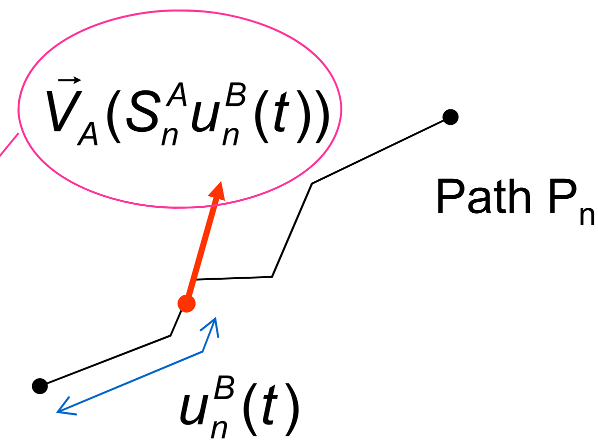
$$E^0(S_0^B) = E^0(S_0^A)$$

Speed twist (3)

B: twisted model



A: original model



$$\vec{V}_B(t) = w(P_n(u_n^B(t))) \times \vec{V}_A(S_n^A u_n^B(t))$$

node location at time t

- At location x, speed is inversely proportional to the target density $b_X(x)$ of location x

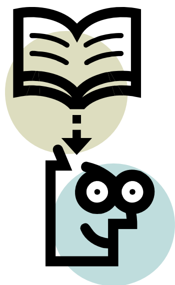
Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?
- Does model accommodate heavy-tailed trip durations ?
- Can model produce a given time-stationary distribution of node position ?

What are mobility data resources ?

Resources

- Partial list:
  - CRAWDAD (crawdad.cs.dartmouth.edu)
  - Haggle (www.haggleproject.org)
  - MobiLib (nile.usc.edu/MobiLib)
  - Street maps:
    - U.S. Census Bureau TIGER database (www2.census.gov/geo/tiger)
    - Mapinfo (www.mapinfo.com)



Frequently Asked Questions
» Summary «

- Power-law inter-contact times are captured by some random trip models
- Trip duration can be heavy tailed
- Given time-stationary distribution of node position can be achieved

Outline

- Simulation Issues with mobility models
- Random trip basic constructs

- A technical condition: Positive Harris recurrence
- Stability of random trip model
- Palm calculus instant primer
- Time-stationary distributions
- Perfect simulation
- FAQ

Appendix 2: An additional condition

- We introduce an additional condition that is needed for stability result of random trip to be valid
  - Positive Harris recurrence
- $Y_n = (I_n, P_n)$ (phase, path) is a Markov chain by construction of the random trip model
  - In general, on general state space !
  - Not necessarily bounded or countable

- We assume that Y_n is positive Harris recurrent
- We check the condition for our catalogue of models

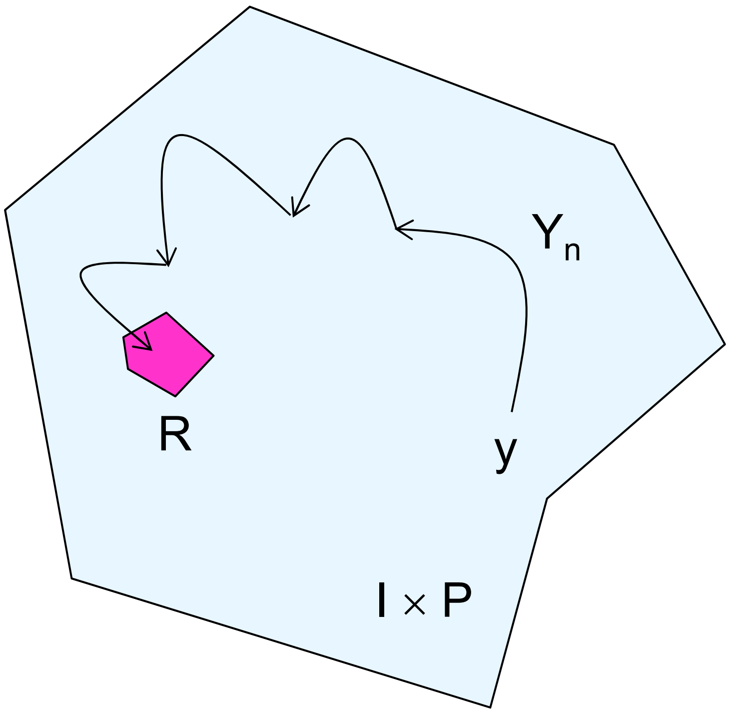
Positive Harris recurrence

- If the state space for the Markov chain of phases and paths *sampled at trip endtimes* would be countable (not true in general), this would mean
  - Any state can be reached
  - No escape to infinity

- A natural condition if we want the mobility state to have a stationary regime

- On a general state space, the definition is more evolved. It is *true for all the models in the catalog before*, assuming common sense assumptions:
  - The underlying graphs are fully connected.
  - Expected number of consecutive visits in a subdomain if finite
  - For billiards, assume density of speed vector is completely symmetric

Harris recurrence

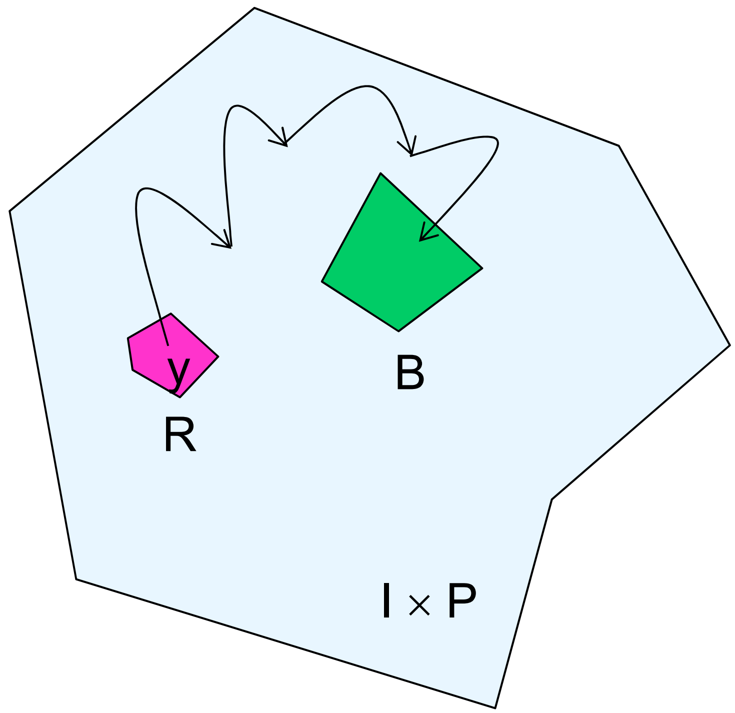


- It means that there exists a set R that is visited by Y_n from any initial state in some given number of transitions

- The set R is "recurrent"

plus …

Harris recurrence (2)



- Probability that Y_n hits a set B starting from R in some given number of transitions is lower bounded by $\beta\ \varphi(B)$
  - β is a number in $(0,1)$, φ is a probability measure on I x P

- The set R is “regenerative”

Positive Harris recurrence

- Y_n Harris recurrent implies that Y_n has a stationary measure π^0 on $I \times P$
  - It may be $\pi^0(I \times P) = +\infty$
- We need $\pi^0(I \times P) < +\infty$ so that Y_n has a stationary probability distribution
- We assume that Y_n is positive Harris recurrent
  - It means Harris recurrent plus that the return time to set R has a finite expectation

Check the condition for random waypoint

- For this model, it is easy

- It suffices to consider RWP with no pauses

- Note that any two paths P_n, P_m such that $|n - m| > 1$ are independent

- Hence

$$P(P_n \in A_1 \times A_2 \mid P_0 = p) = |A_1| \cdot |A_2|, \text{ for all } n > 1$$

- Take as the recurrent set $R \equiv A \times A$

Check condition for restricted random waypoint

The condition is true if

- In addition to assumptions for random waypoint, it holds
  - The Markov walk on sub-domains is irreducible
  - And the mean number of trips within a sub-domain is finite

- Proof follows from well known stability results for Markov chains on finite state spaces

Check condition for random walk on torus 

The condition is true if

- The speed vector has a density in R^2
- And, trip duration has a density, conditional on either phase is move or pause

Check condition for random walk on torus(2)

- Main thing to prove is that node position at trip transitions, X_n, is Harris recurrent

- Fact: the distribution of X_n started from any given initial point, converges to uniform distribution, provided *only* that node speed *has a density*

- Harris recurrence follows by the latter fact, Erdos-Turan-Koksma inequality, and Fourier analysis

Check condition for billiards 

The condition is true if

- The speed vector has a density in R^2 that is *completely symmetric*
- And, trip duration has a density, conditional on either phase is move or pause

- Proof by reduction to random walk (see [LV06])

- Def. A random vector (X,Y) is said to have a completely symmetric distribution iff (-X,Y) and (X,-Y) have the same distribution as (X,Y)

To be complete …

- We also need to assume:

  (a) Trip duration S_n is strictly positive
  (b) Distribution of trip duration S_n is non-arithmetic

  arithmetic = on a lattice

- These are minor conditions, can in practice be assumed to hold
  - (a) is common sense
  - (b) is true in particular if S_n has a density