SATISFIABILITY OF ELASTIC DEMAND IN THE SMART GRID

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EPFL

Energy Systems Day
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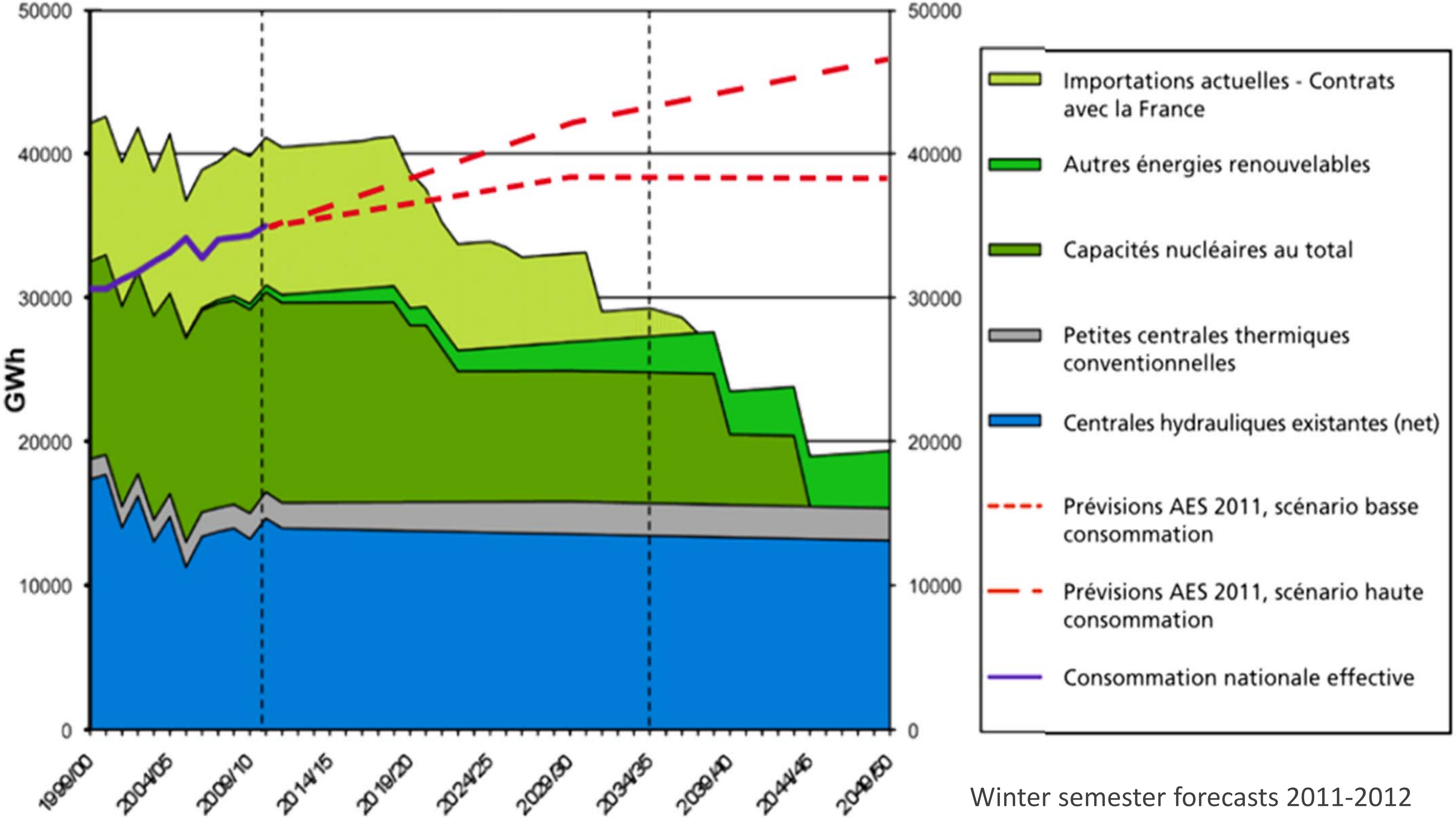
Cambridge, 2012 March 12

Contents

Swiss Future Supply is Uncertain



Winter semester forecasts 2011-2012

- Source: ASE (Association des Entreprises Electriques Suisses,)

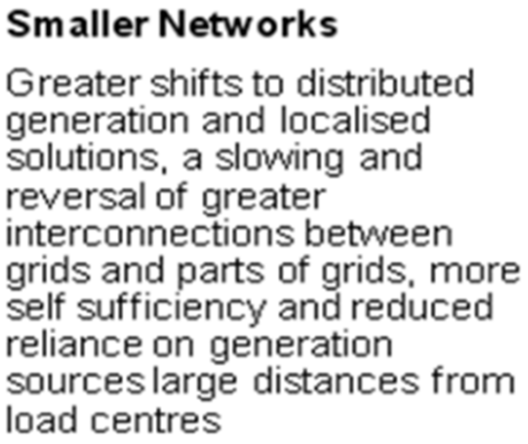
Smart Grid Vision : Larger Network

Larger Networks

A shift towards larger and larger aggregated networks and new, major interconnections between large load and generation centres over long distances (including continents)

Source:
ELECTRICITY SUPPLY SYSTEMS OF THE FUTURE
White paper on behalf of the CIGRE Technical Committee
TC Chair: Klaus Froelich
2011

Smart Grid Vision : Smaller, More Autonomous Networks



Smaller Networks

Greater shifts to distributed generation and localised solutions, a slowing and reversal of greater interconnections between grids and parts of grids, more self sufficiency and reduced reliance on generation sources large distances from load centres

Source:
ELECTRICITY SUPPLY SYSTEMS OF THE FUTURE
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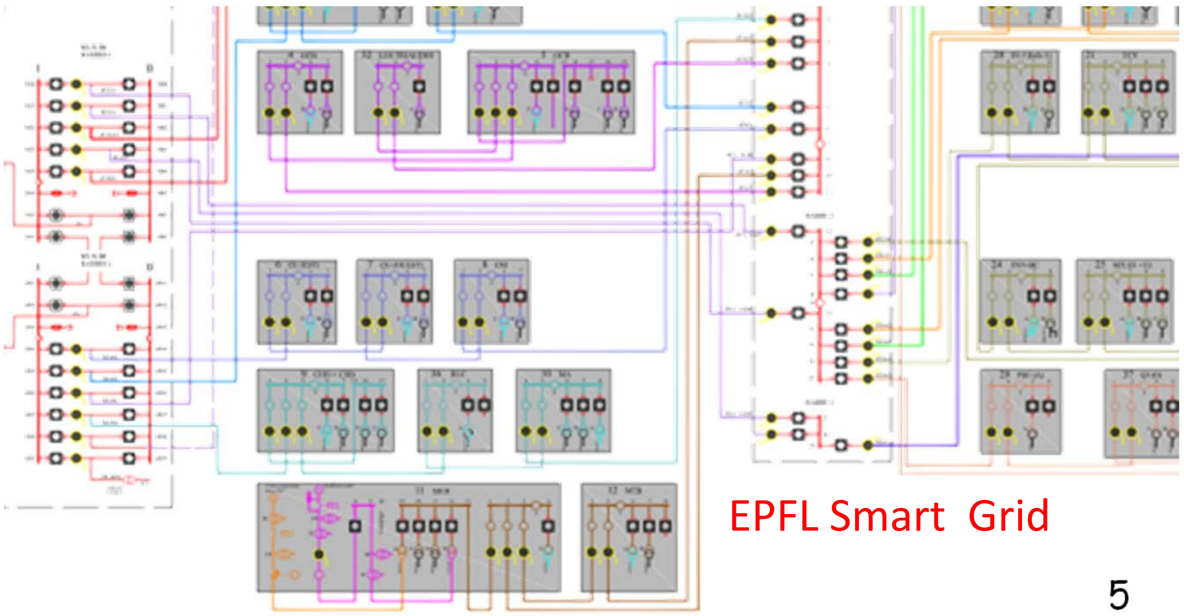
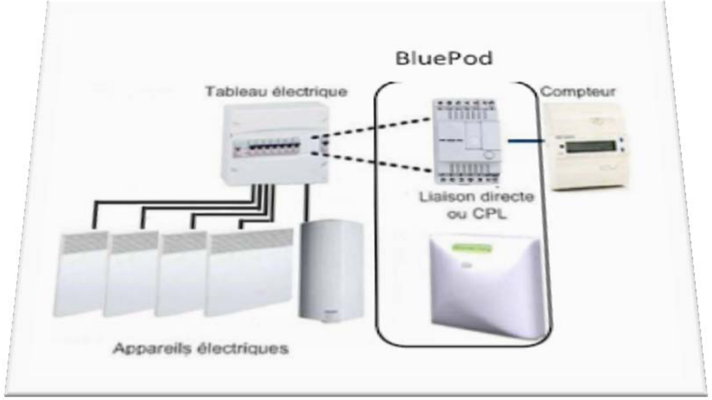
- Flexible services as alternative to blackout, including across slack bus
  - Islanded operation possible



EPFL Smart Grid

Flexible Services

- Flexible services = distribution network operator may interrupt / modulate power
- elastic loads support graceful degradation
- Thermal load (Voltalis), washing machines (Romande Energie«commande centralisée»)
  e-cars,



Voltalis Bluepod switches off thermal load for 60 mn

Our Problem Statement

- Do elastic services work ?
  - Delays
  - Returning load
- **Problem Statement**
  Is there a control mechanism that can stabilize demand ?
- A very course (but fundamental) first step
- We leave out for now the details of signals and algorithms

2.

A MODEL OF ELASTIC DEMAND

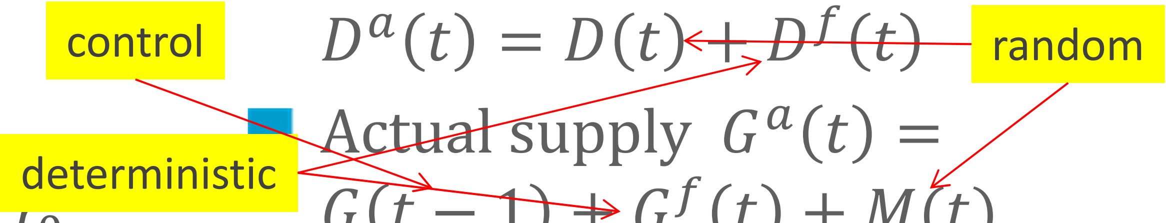
Macroscopic Model of Cho and Meyn [1], non elastic demand, mapped to discrete time

Step 1: Day-ahead market

- Forecast demand: $D^f(t)$
- Forecast supply: $G^f(t) = D^f(t) + r_0$

Step 2: Real-time market

- Actual demand $D^a(t) = D(t) + D^f(t)$
- Actual supply $G^a(t) = G(t-1) + G^f(t) + M(t)$



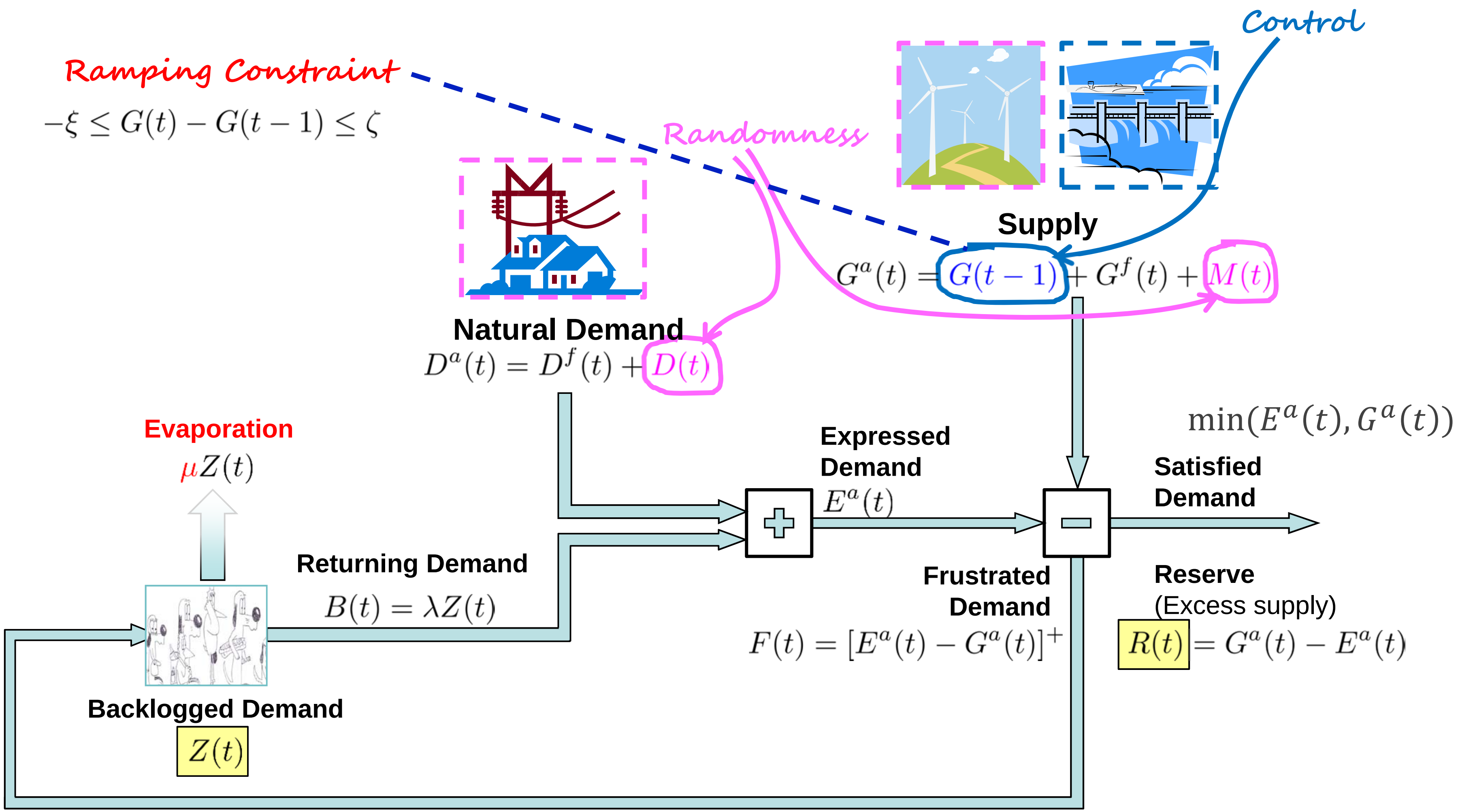
control

deterministic

random

- We now add the effect of elastic demand / flexible service
  Some demand can be «frustrated» (delayed)

Our Macroscopic Model with Elastic Demand



Ramping Constraint

$-\xi \leq G(t) - G(t-1) \leq \zeta$

Control

Randomness

Supply

$G^a(t) = G(t-1) + G^f(t) + M(t)$

Natural Demand

$D^a(t) = D^f(t) + D(t)$

Evaporation

$\mu Z(t)$

$\min(E^a(t), G^a(t))$

Expressed Demand

$E^a(t)$

Satisfied Demand

Returning Demand

$B(t) = \lambda Z(t)$

Frustrated Demand

$F(t) = [E^a(t) - G^a(t)]^+$

Reserve

(Excess supply)

$R(t) = G^a(t) - E^a(t)$

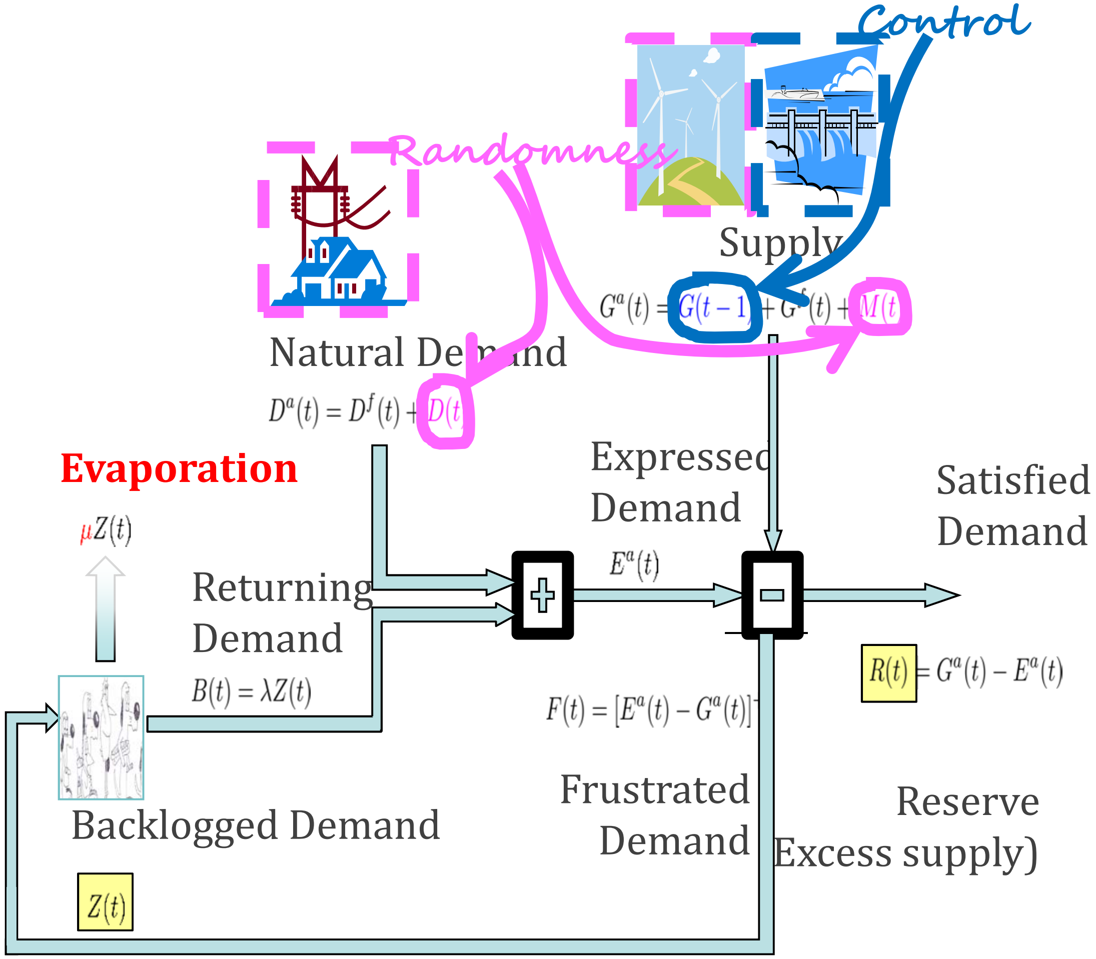
Backlogged Demand

$Z(t)$

$$R(t) = G(t-1) - \lambda Z(t) + M(t) - D(t) + r_0$$
$$Z(t) = Z(t-1) - \lambda Z(t) - \mu Z(t) + \mathbb{1}_{\{R(t)<0\}} |R(t)|$$

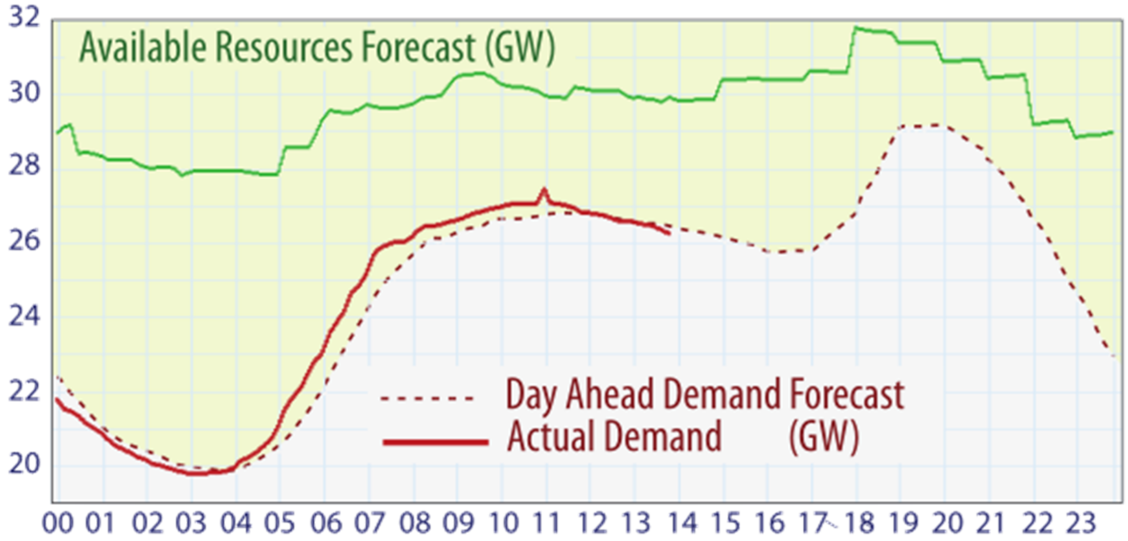
Backlogged Demand

- We assume backlogged demand is subject to two processes: update and re-submit
- Update term (evaporation): $\mu Z\ dt$ with $\mu > 0$ or $\mu < 0$ μ is the evaporation rate (proportion lost per time slot)
- Re-submission term $\lambda Z\ dt$ $1/\lambda$ (time slots) is the average delay



Macroscopic Model, continued

- Assumption : $(M - D) = \mathrm{ARIMA}(0, 1, 0)$
  typical for deviation from forecast
  $\big(M(t+1) - D(t+1) - M(t) - D(t)\big) := N(t+1) \sim N(0, \sigma^2)$



S. Meyn
"Dynamic Models and Dynamic Markets
for Electric Power Markets"

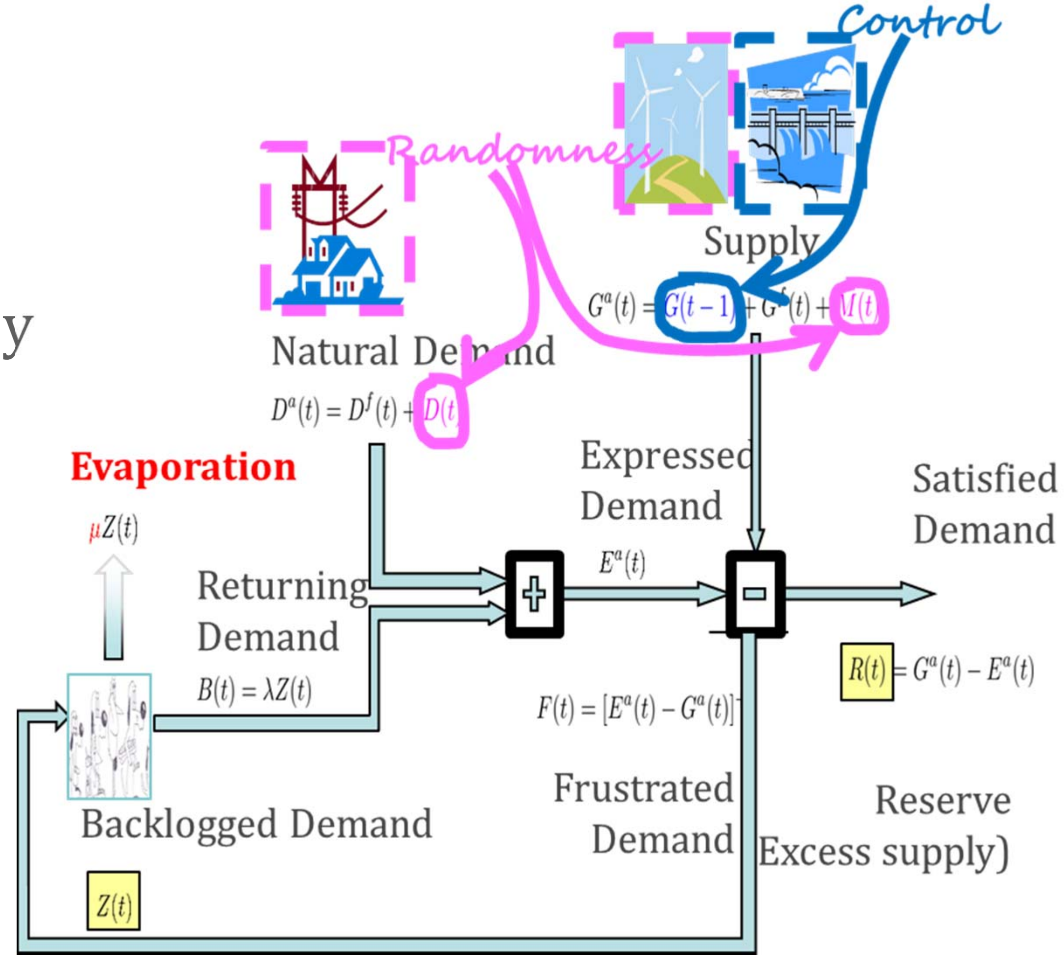
- 2-d Markov chain on continuous state space

$$
\begin{aligned}
R(t+1) &= R(t) + \Delta G(t) + N(t+1) - \lambda[Z(t+1) - Z(t)] \\
Z(t+1) &= (1 - \lambda - \mu) Z(t) + \mathbb{1}_{\{R(t)<0\}} R(t)
\end{aligned}
$$

The Control Problem

- **Control variable:**
  $G(t-1)$
  production bought one time slot ago in real time market
- Controller sees only supply $G^a(t)$ and expressed demand $E^a(t)$
- **Our Problem:**
  keep backlog $Z(t)$ stable
- Ramp-up and ramp-down constraints

  $\xi \leq G(t) - G(t-1) \leq \zeta$



Threshold Based Policies

$$G^f(t) = D^f(t) + r_0$$

Forecast supply is adjusted to forecast demand

$$R(t) = G^a(t) - E^a(t)$$

R(t) := reserve = excess of demand over supply

Threshold policy:

if $R(t) < r*$ increase supply to come as close to r^*as possible (considering ramp up constraint)

else decrease supply to come as close to r^*as possible (considering ramp down constraint)

Simulation

- Linearized system: 1 is eigenvalue

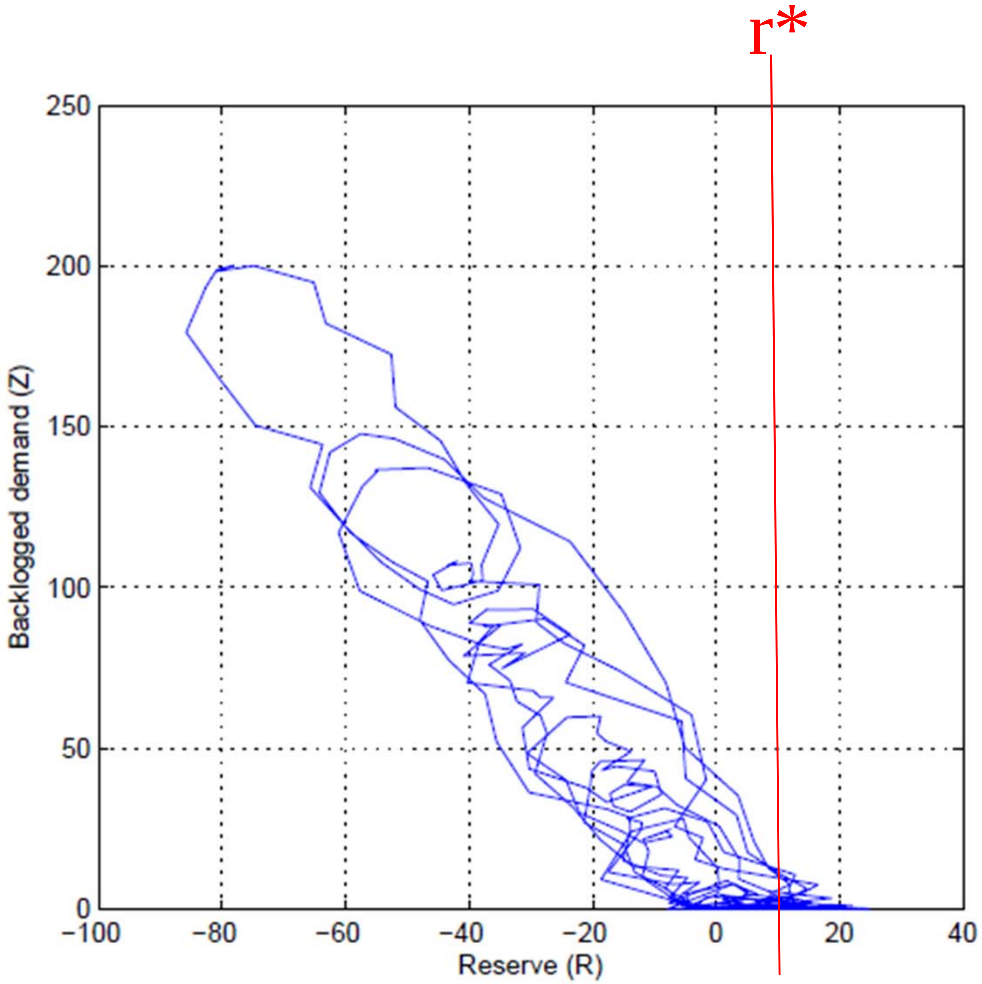


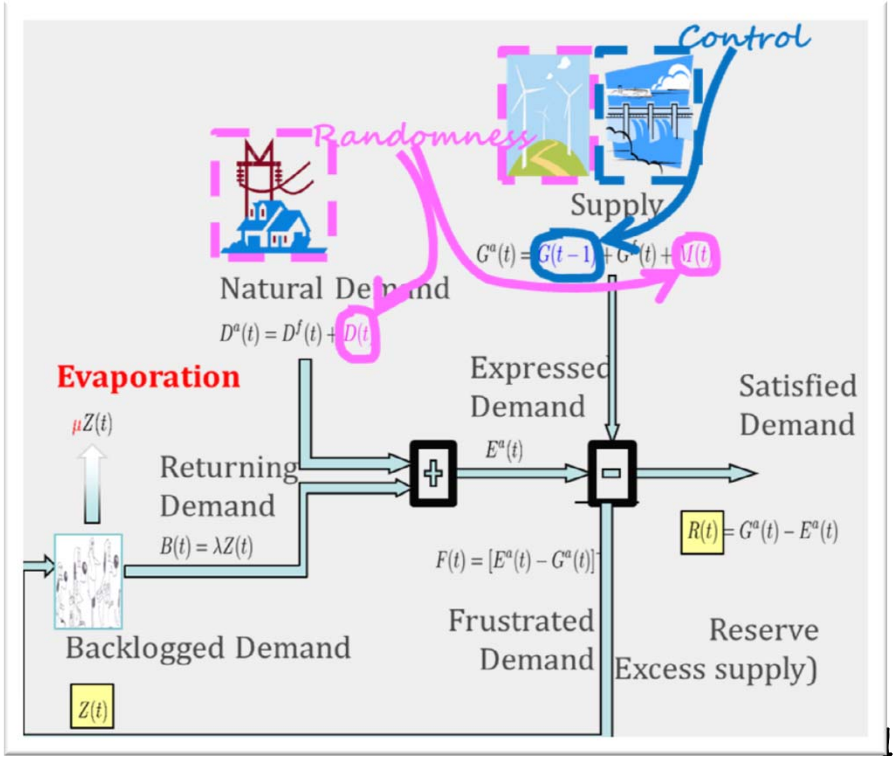
Figure 2. 500 iterations of the Markov process (13)-(14) for $\zeta = 1, r^* = 10, \sigma = 5, \lambda = 0.3, \mu = 0.1$

3.

STABILITY RESULTS

Findings

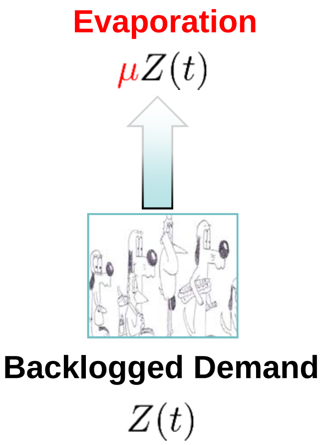
- If evaporation μ is positive, system is stable (ergodic, positive recurrent Markov chain) for any threshold $r*$
- If evaporation μ is negative, system unstable for any threshold $r*$
- Delay does not play a role in stability
- Nor do ramp-up / ramp down constraints or size of reserve



More Detailed Findings

- Case 1: $\mu > 0$
  Postponing a task = discount

- **Theorem 1**: The Markov chain *(R,Z)* is Harris recurrent and ergodic. It converges to the unique steady state probability distribution, for *any threshold and any strictly positive ramp-up constraint*.



Evaporation

$\mu Z(t)$

Backlogged Demand

$Z(t)$

- Case 2: $\mu < 0$
  Postponing a task = penalty

- **Theorem 2**: The Markov chain *(R,Z)* is non-positive, for *any threshold*.

Method of Proof: quadratic Lyapunov (case 1) or logarithmic L. (case 2)

Evaporation

- *Negative* evaporation μ means: delaying a load makes the *returning load* larger than the original one.

- Could this happen ?

Q. Does letting your house cool down now imply
spending more heat in total
compared to
keeping temperature constant ?

- $\neq$ return of the load:

Q. Does letting your house cool down now imply spending more heat later ?

A. Yes
(you will need to heat up your house later -- delayed load)

- Assume the house model of [6]

heat provided to building $\boxed{d(t)\epsilon} = \underset{\text{leakiness}}{\boxed{K}}(T(t) - \underset{\text{outside}}{\theta(t)}) + \underset{\text{inertia}}{\boxed{C}}(T(t) - T(t-1))$

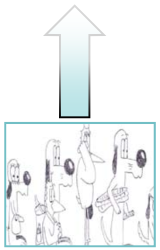
efficiency $\epsilon \underset{\text{E, total energy provided}}{\boxed{\sum_{t=1}^{\tau} d(t)}} = K \sum_{t=1}^{\tau} \left(\underset{\text{achieved t}^{\text{o}}}{\boxed{T(t)}} - \theta(t) \right) + C(T(\tau) - T(0)$

Scenario	***Optimal***	***Frustrated***
Building temperature	$T^*(t), t = 0 \dots \tau$	$T(t), t = 0 \dots \tau,$ $T(t) \leq T^*(t)$
Heat provided	$E^* = \frac{1}{\epsilon}\left(K \sum_{t=1}^{\tau} (T^*(t) - \theta(t)) + C(T^*(\tau) - T^*(0)\ ) \right)$	$E < E^*$

When Delayed Heating is Less Heat

- With constant coefficient of performance ϵ, total energy provided is less if let building cool down and warm up again

- Assume some demand is frustrated (second scenario)
  update process replaces backlogged demand by *what is needed to recover the target temperature T^**



Evaporation

$\mu Z(t)$

Backlogged Demand

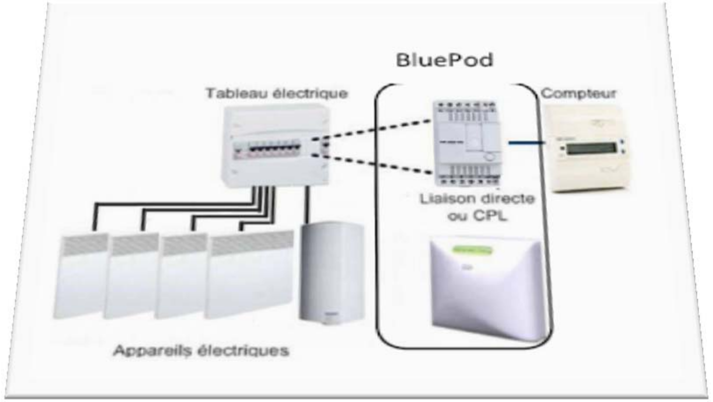
$Z(t)$

- Update process decreases backlog, evaporation is positive

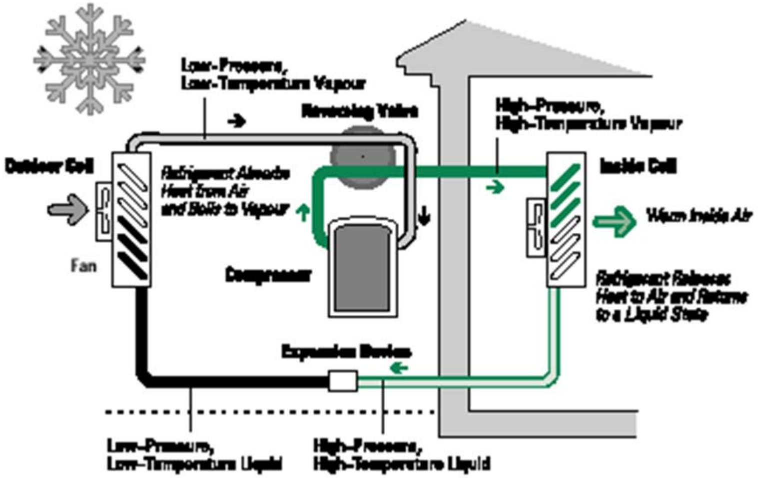
The Sign of Evaporation

- Resistive heating system: evaporation is positive.

  This is why Voltalis bluepod is accepted by users



- If heat = heat pump, coefficient of performance ϵ may be variable negative evaporation is possible



- Electric vehicle: delayed charge may have to be faster, less efficient, negative evaporation is possible

Conclusions

- A first model of adaptive appliances with volatile demand and supply

- Suggests that negative evaporation makes system unstable
  Existing demand-response positive experience (with Voltalis/PeakSaver) might not carry over to other loads

- Model suggests that large backlogs are possible
  Backlogged load is a new threat to grid operation
  Need to measure and forecast backlogged load

Questions ?

[1] Cho, Meyn – *Efficiency and marginal cost pricing in dynamic competitive markets with friction,* Theoretical Economics, 2010

[2] Le Boudec, Tomozei, *Satisfiability of Elastic Demand in the Smart Grid,* Energy 2011 and ArXiv.1011.5606

[3] Le Boudec, Tomozei, *Demand Response Using Service Curves,* IEEE ISGT-EUROPE, 2011

[4] Le Boudec, Tomozei, *A Demand-Response Calculus with Perfect Batteries*, WoNeCa, 2012

[5] Papavasiliou, Oren - *Integration of Contracted Renewable Energy and Spot Market Supply to Serve Flexible Loads,* 18th World Congress of the International Federation of Automatic Control, 2011

[6] David MacKay, *Sustainable Energy – Without the Hot Air,* UIT Cambridge, 2009