SATISFIABILITY OF ELASTIC DEMAND IN THE SMART GRID

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ÉCOLE POLYTECHNIQUE
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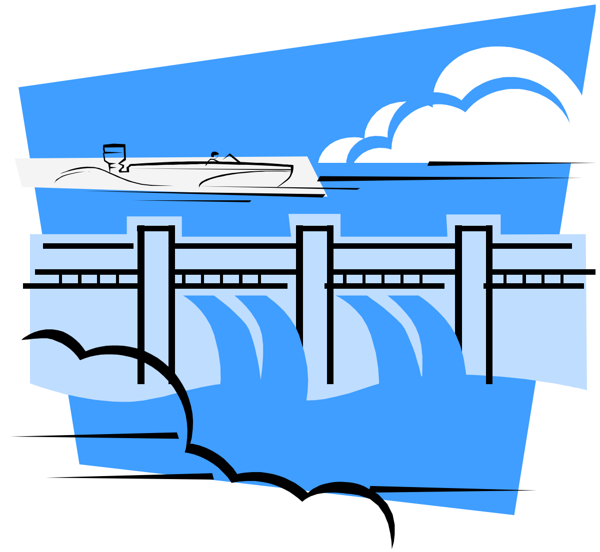
Demand Management

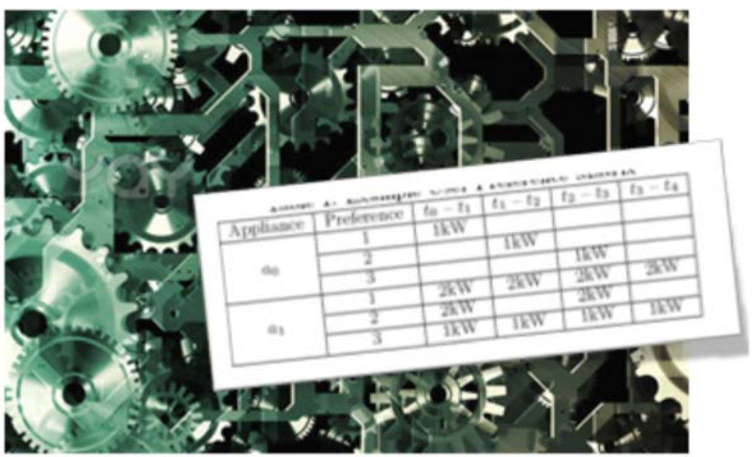


- Variable Supply
- Variable Demand
- Frequency Response becomes insufficient
- Demand management required in future grid

Demand Management must Be Simple, Adaptive and Distributed

- Global, optimal schedules
- But they are
  - inflexible
  - complex

- *Managing End-User Preferences in the Smart Grid*, C. Wang and M. d. Groot, E-energy 2010, Passau, Germany, 2010

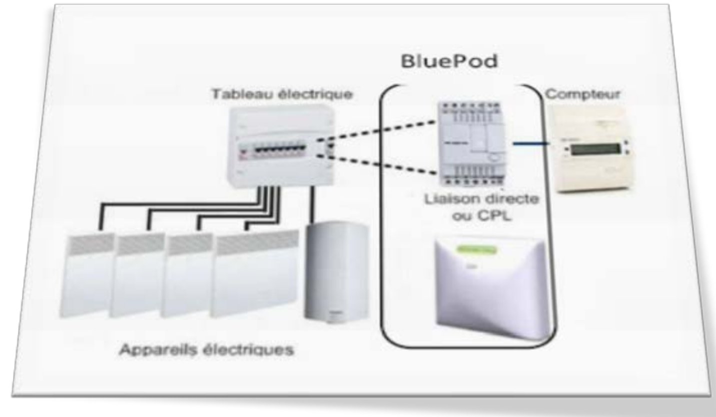


Appliance	Preference	$f_0 - f_1$	$f_1 - f_2$	$f_2 - f_3$	$f_3 - f_4$
a_0	1	1kW			
	2		1kW		
	3			1kW	
a_1	1	2kW	2kW	2kW	2kW
	2	2kW		2kW	
	3	1kW	1kW	1kW	1kW

Home automation controller hung yesterday night. Hot water was not replenished overnight.

Adaptive Appliances

- Some Demand can be delayed !
- DSO provides best effort service with statistical guarantees [Keshav and Rosenberg 2010]



Voltalis Bluepod switches off thermal load for 30 mn

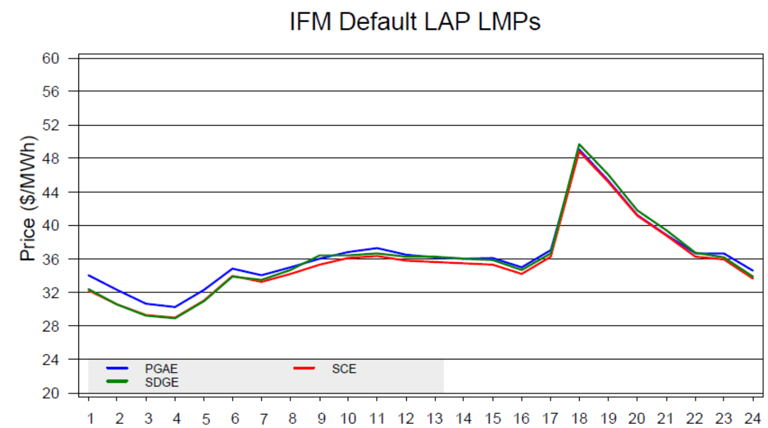
Programmable dishwasher

PeakSaver cycles AC for 15mn

Our Problem Statement

- Is elastic demand feasible ?
- We leave out (for now) the details of signals and algorithms
- **Problem Statement**
  Is there a control mechanism that can stabilize demand ?
- Instability can be generated by
  - Delays in demand
  - Higher returning demand
- A very course (but fundamental) first step

Macroscopic Model



Step 1: Day-ahead market

- Forecast demand : $D_f(t)$
- Forecast supply : $G_f(t) = D_f(t) + r_0$

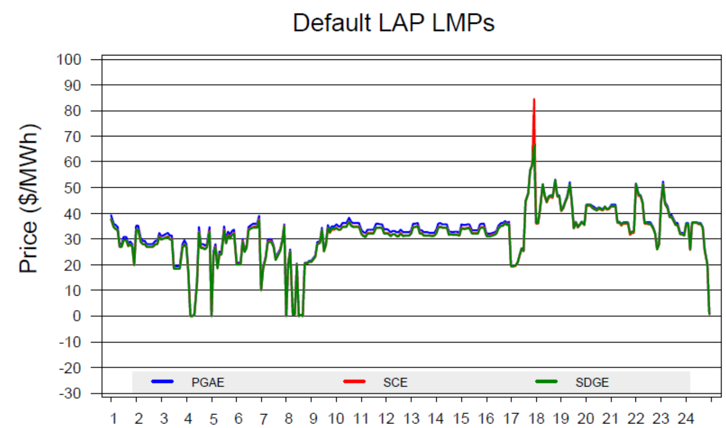
Step 2: Real-time market

- Actual demand : $D_a(t) = D(t) + D_f(t)$
- Actual supply : $G_a(t) = G(t-1) + G_f(t) + M(t)$

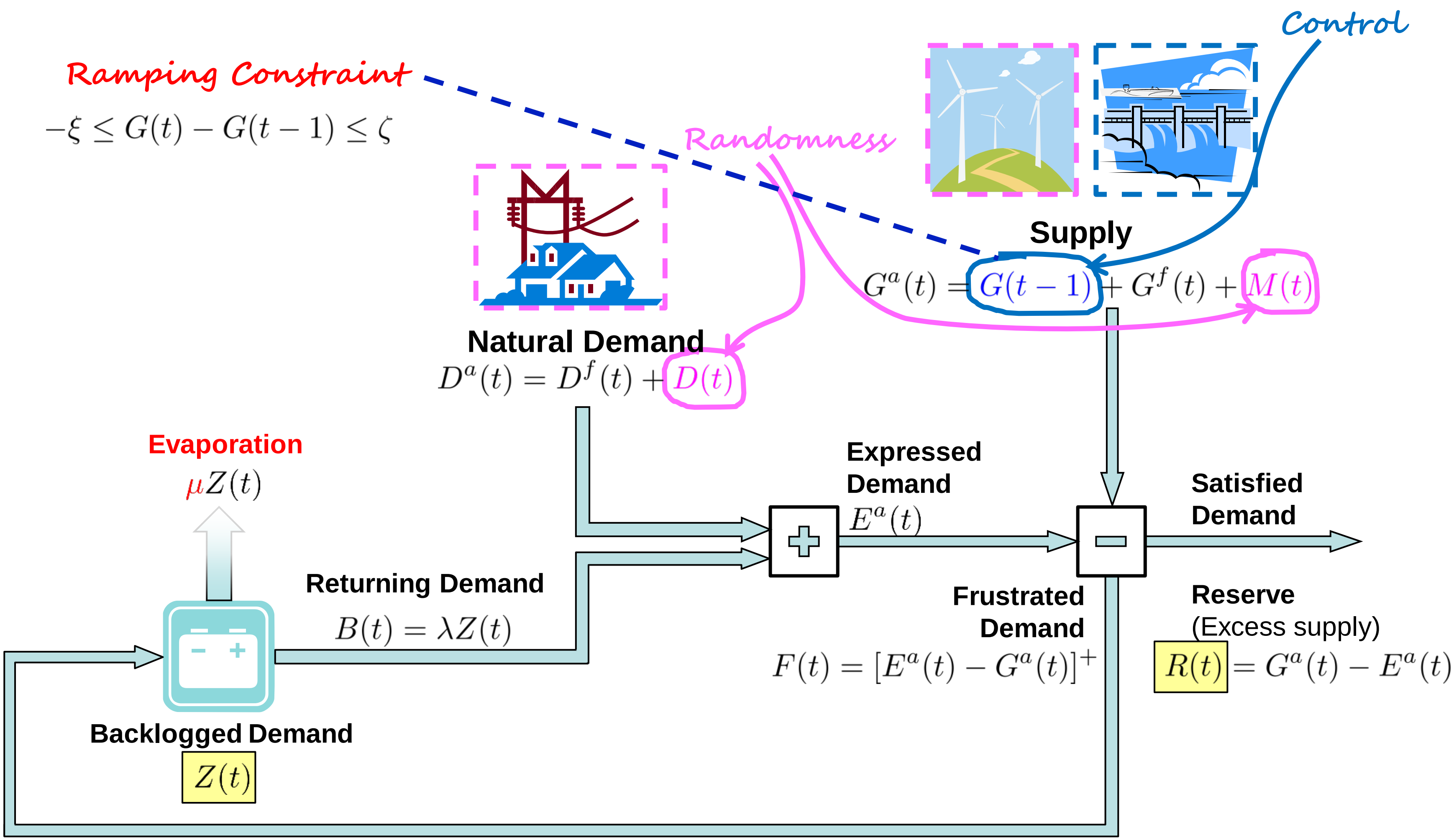
- Deterministic processes : $D_f(t), G_f(t)$

- Random processes : $M(t), D(t)$

- Control : $G(t-1)$
  - Ramp up, ramp down



Macroscopic Model



$$R(t) = G(t-1) - \lambda Z(t) + M(t) - D(t) + r_0$$
$$Z(t) = Z(t-1) - \lambda Z(t) - \mu Z(t) + \mathbb{1}_{\{R(t)<0\}} |R(t)|$$

Macroscopic Model – Normal Assumption

- Assumption : (M – D) = ARIMA(0, 1, 0)

$$(M(t+1) - D(t+1)) - (M(t) - D(t)) = N(t+1) \sim \mathcal{N}(0, \sigma^2)$$

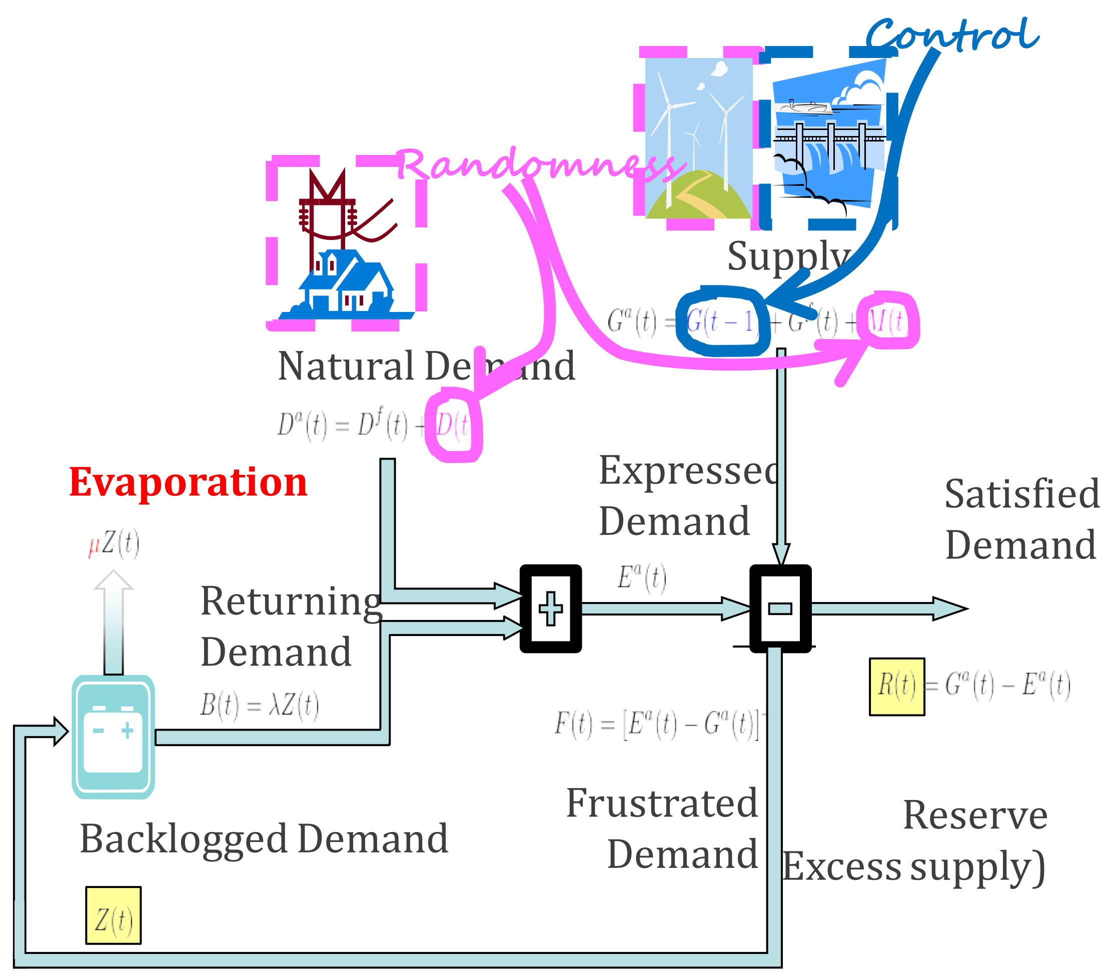
- 2-d Markov chain on continuous state space

$$R(t+1) = R(t) + \Delta G(t) + N(t+1) - \lambda[Z(t+1) - Z(t)]$$
$$Z(t+1) = (1 - \lambda - \mu)Z(t) + \mathbb{1}_{\{R(t)<0\}}R(t)$$

The Control Problem

- **Control variable:**
  G(t-1)
  production bought one second ago in real time market
- Controller sees only supply $G^a(t)$ and expressed demand $E^a(t)$
- **Our Problem:**
  keep backlog Z(t) stable
- Ramp-up and ramp-down constraints

  $\xi \leq G(t) - G(t-1) \leq \zeta$



Threshold Based Policies

$$G^f(t) = D^f(t) + r_0$$

Forecast supply is adjusted to forecast demand

$$R(t) = G^a(t) - E^a(t)$$

R(t) := reserve = excess of demand over supply

Threshold policy:

if R(t) < r* increase supply as much as possible (considering ramp up constraint)

else set R(t)=r*

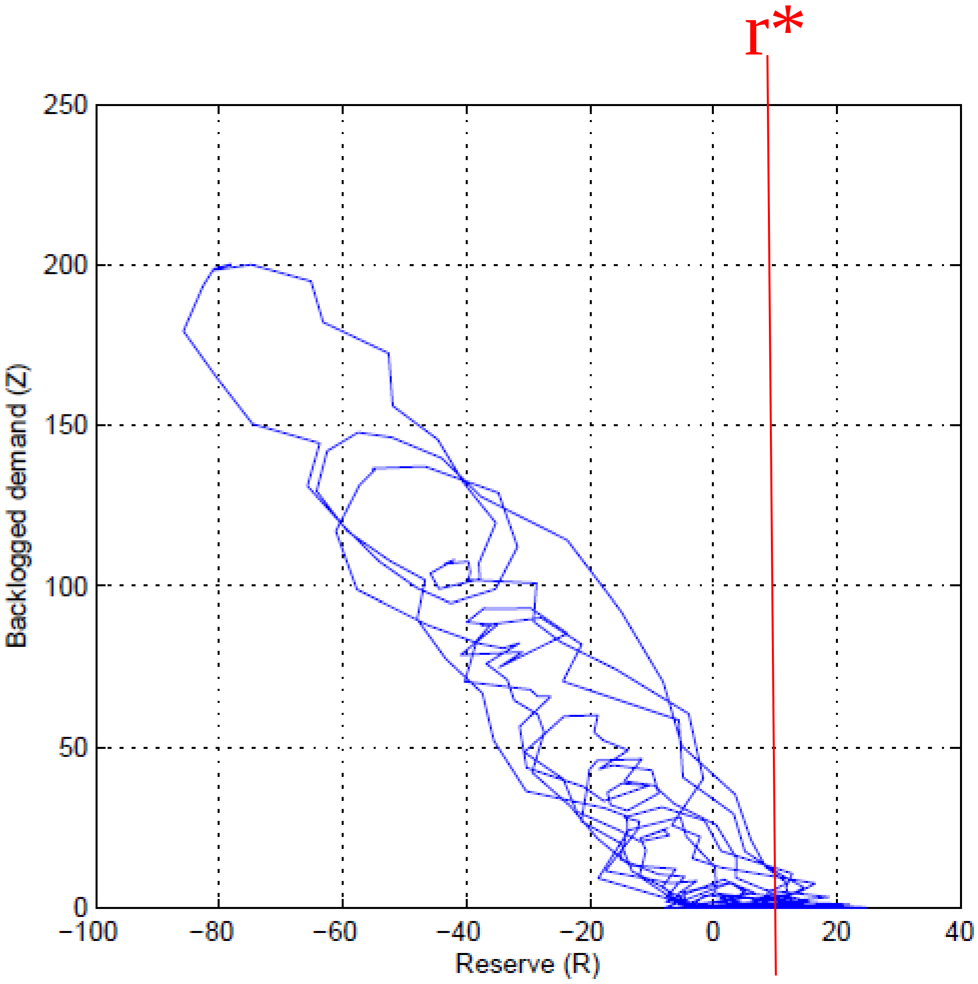
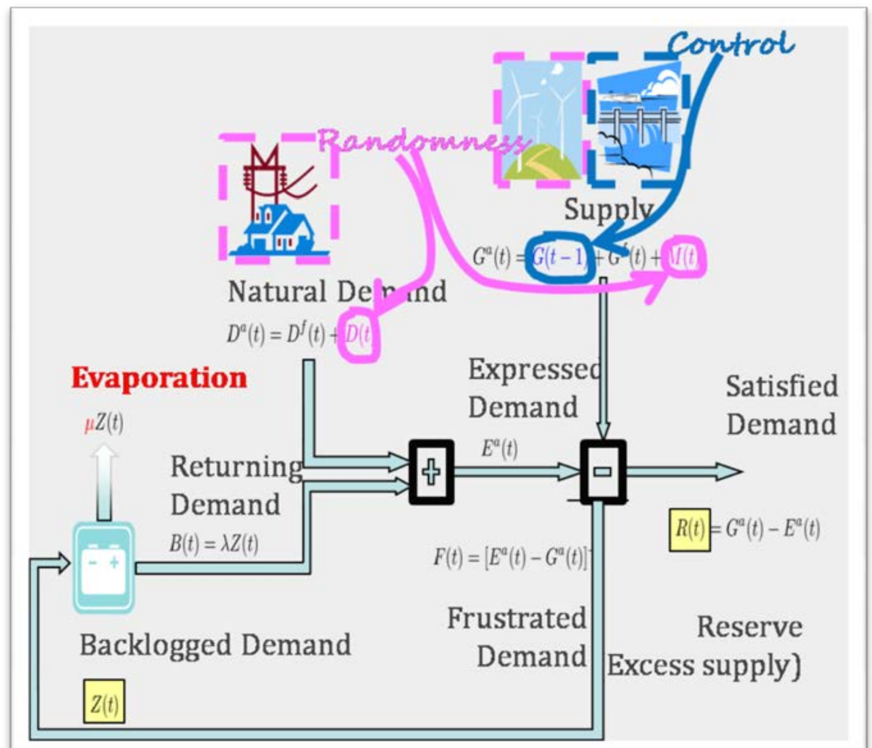


Figure 2. 500 iterations of the Markov process (13)-(14) for $\zeta = 1, r^* = 10, \sigma = 5, \lambda = 0.3, \mu = 0.1$

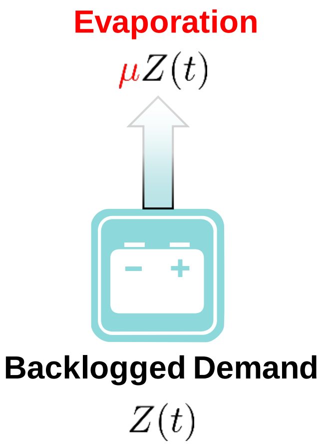
Findings

- If evaporation μ is positive, system is stable (ergodic, positive recurrent Markov chain) for any threshold r*
- If evaporation is negative, system unstable for any threshold r*
- Delay does not play a role in stability
- Nor do ramp-up / ramp down constraints or size of reserve



More Detailed Findings

- Case 1: Positive evaporation
  Postponing a task = discount



■ **Theorem 1**: The Markov chain *(R,Z)* is Harris recurrent and ergodic. It converges to the unique steady state probability distribution, for *any threshold and any strictly positive ramp-up constraint*.

- Case 2: Negative evaporation
  Postponing a task = penalty

■ **Theorem 2**: The Markov chain *(R,Z)* is non-positive, for *any threshold*.

Method of Proof: quadratic Lyapunov (case 1) or logarithmic L. (case 2)

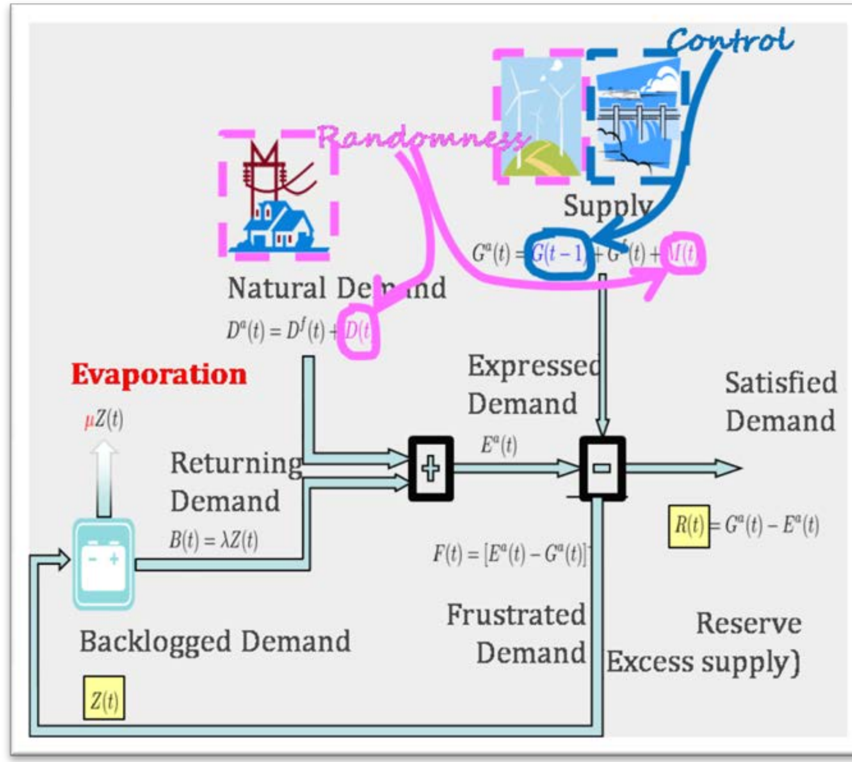
Evaporation

- Evaporation = dropped fraction of delayed demand
- *Negative* evaporation means:

  delaying a demand makes the *returning demand* larger than the original one

- Could this happen ?

  Does letting your house cool down now implies spending more heat later ? (vs keeping constant temperature)



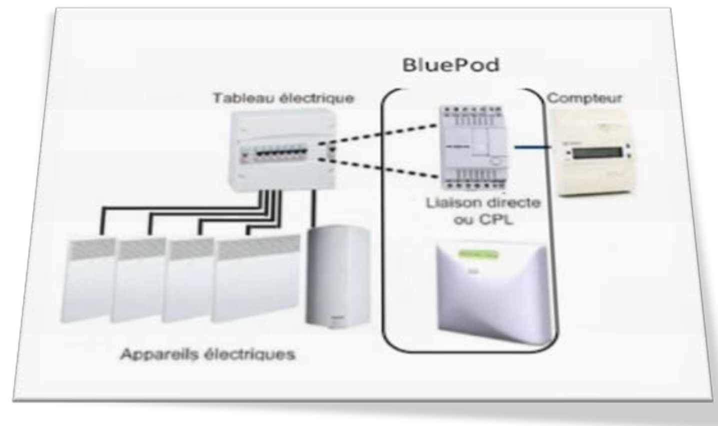
- Do not confuse with the sum of returning demand + current demand, which is always larger than current demand)

Evaporation: Heating Appliances

- Assume the house model of [MacKay 2009]

heat provided to building $d(t)\epsilon = \boxed{K}(T(t) - \theta(t)) + \boxed{C}(T(t) - T(t-1))$

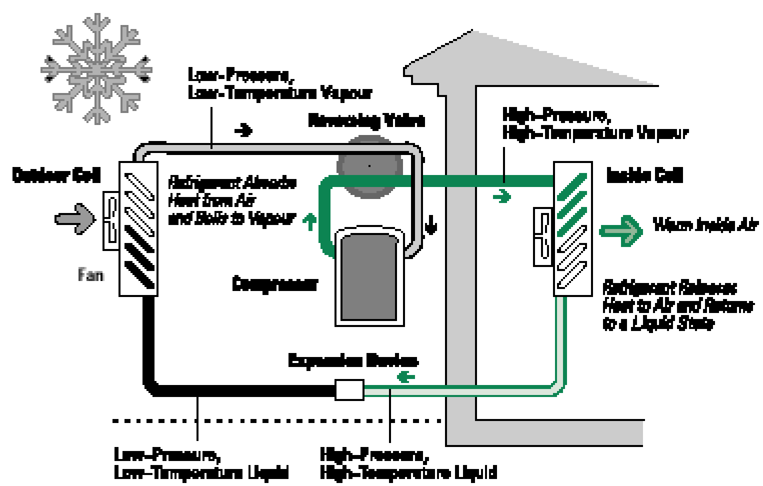
leakiness outside inertia



then delayed heating is less heating

- If heat = energy, then evaporation is positive.

  This is why Voltalis bluepod is accepted by users



- If heat = heat pump, coefficient of performance may be variable
  Delayed heating with air heat pump may have negative evaporation

Batteries

- Thermal loss is non linear, delayed loading causes negative evaporation

  (charging at higher intensity)

Conclusions

- A first model of adaptive appliances with volatile demand and supply
- Suggests that negative evaporation makes system unstable,
  - thus detailed analysis is required to avoid it
- Model can be used to quantify more detailed quantities
  - E.g. amount of backlog, optimal reserve

Questions ?

[1] Cho, Meyn – *Efficiency and marginal cost pricing in dynamic competitive markets with friction*

[2] Papavasiliou, Oren - *Integration of Contracted Renewable Energy and Spot Market Supply to Serve Flexible Loads*

[3] Operational Requirements and Generation Fleet Capability at 20% RPS (CAISO - 31 August, 2010)