

# Real-Time Control of Electrical Distribution Grids

Jean-Yves Le Boudec<sup>1,2</sup>

EPFL

NREL

2018 March 23

<sup>1</sup> <https://people.epfl.ch/105633/research>

<sup>2</sup> <http://smartgrid.epfl.ch>

# Credits

## ***Joint work***

EPFL-DESL (Electrical Engineering)  
and LCA2 (Computer Science)

## ***Supported by***



**Energy Turnaround**  
National Research Programme NRP 70



## ***Contributors***

Jagdish Acharya  
Andrey Bernstein  
Niek Bouman  
Benoit Cathiard  
Andreas Kettner  
Maaz Mohiuddin  
Mario Paolone  
Marco Pignati  
Lorenzo Reyes  
Roman Rudnik  
Erica Scolari  
Wajeb Saab  
Cong Wang

# Contents

1. Real-time operation of distribution grids
2. V-control

# 1. Real-Time Operation of Microgrid: Motivation

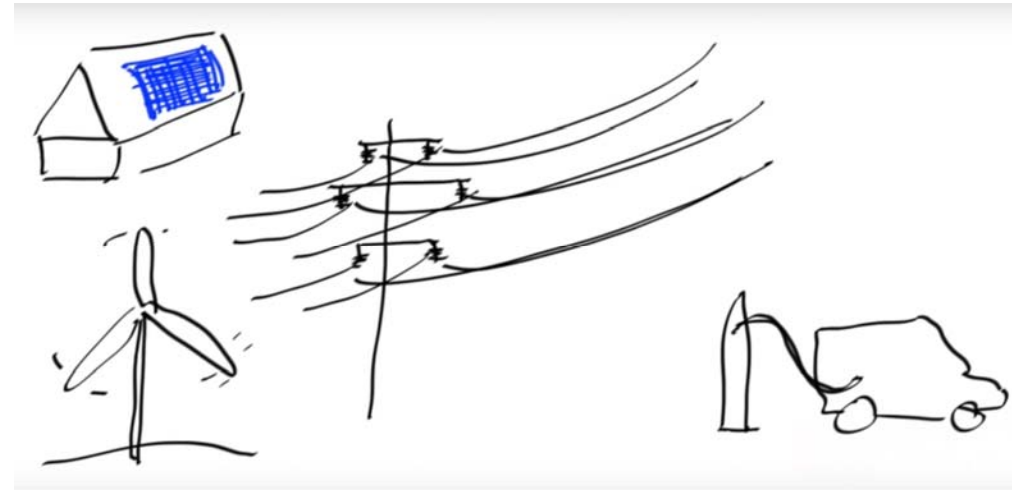
Absence of inertia (inverters)

Stochastic generation (PV)

Storage, demand response

Grid stress (charging stations,  
heat pumps)

Support main grid (frequency, AGC)

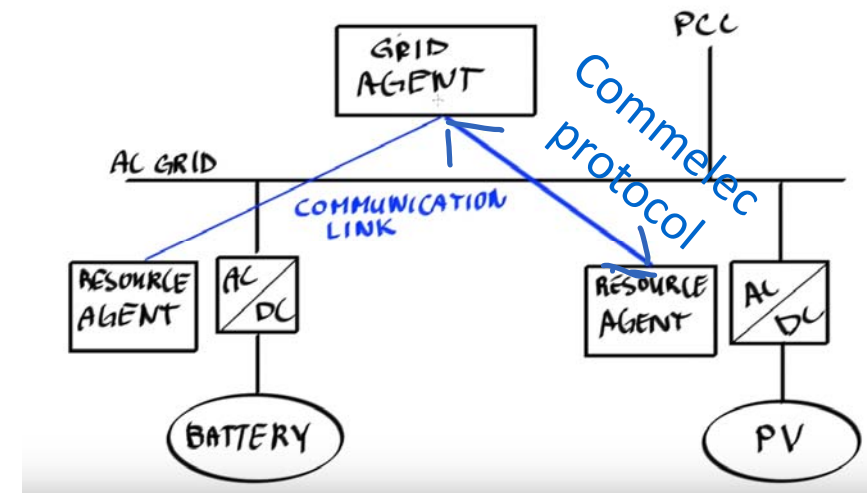


⇒ Agent based, real-time control of microgrid

# COMMELEC Uses Explicit Power Setpoints

Grid Agent = software agent,  
manages grid, uses PMUs

Resource Agent = software agent,  
manages device



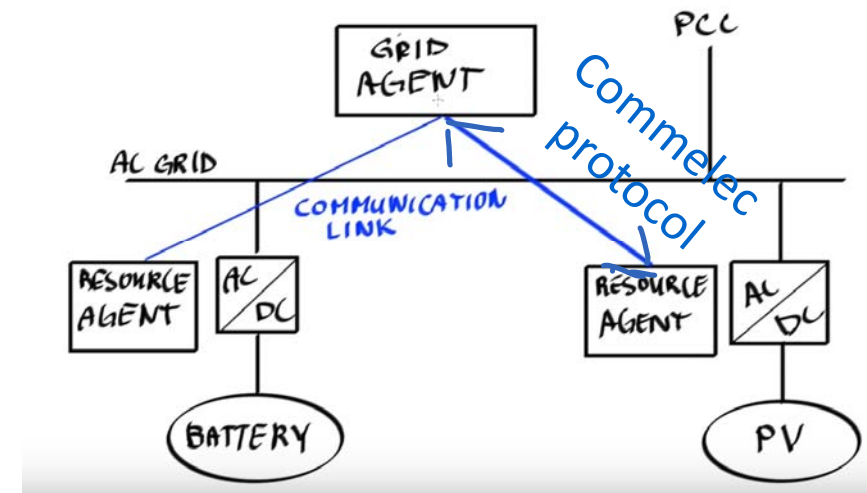
Grid Agent sends **explicit power setpoints** to Resource Agents

Goal: manage quality of service in grid; support main grid; use resources optimally. [Bernstein et al 2015, Reyes et al 2015]

# COMMELEC Principle of Operation

Every 100 msec

- Resource agent sends to grid agent:  
PQ profile, Virtual Cost and  
**Belief Function**
- Grid agent sends power setpoints



PQ profile = set of setpoints that this resource is willing to receive

Virtual cost = cost attached to receiving a setpoint

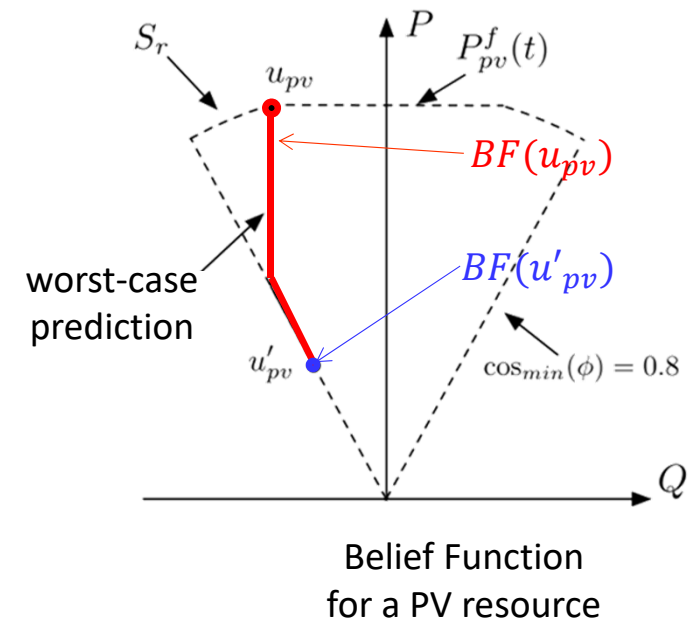
# Belief Function

Say grid agent requests setpoint  $(P_{\text{set}}, Q_{\text{set}})$  from a resource; actual setpoint  $(P, Q)$  will, in general, differ.

**Belief function** exported by resource agent means: the resource implements  $(P, Q) \in BF(P_{\text{set}}, Q_{\text{set}})$

Quantifies uncertainty

Essential for safe operation



# Operation of Grid Agent

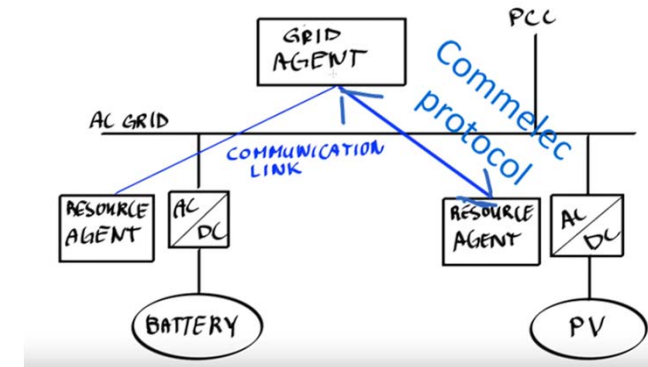
Grid agent computes a setpoint vector  $x$  that minimizes

$$J(x) = \sum_i w_i C_i(x_i) + W(z) + J_0(x_0)$$

Virtual cost of the resources      
 Penalty function of grid electrical state  $z$   
(e.g., voltages close to 1 p.u., line currents below the ampacity)      
 Cost of power flow at point of common connection

subject to **admissibility**.

$x$  is admissible  $\Leftrightarrow (\forall x' \in BF(x), \quad x' \text{ satisfies security constraints})$



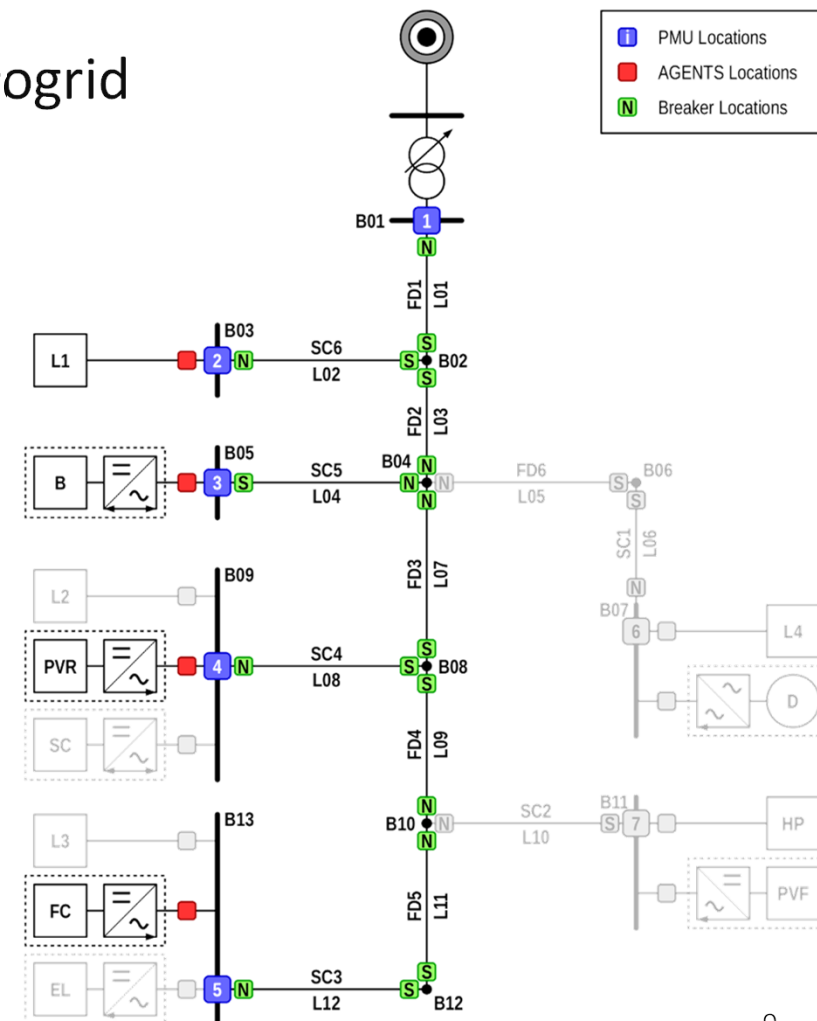


# Implementation / EPFL Microgrid

Topology: 1:1 scale of the Cigré low-voltage microgrid benchmark TF C6.04.02 [Reyes et al, 2018]

- Phasor Measurement Units:  
nodal voltage/current syncrophasors
- Phasor Data Concentrator
- Discrete Kalman Filter State estimator
- PVs, Battery, Load (flex house)

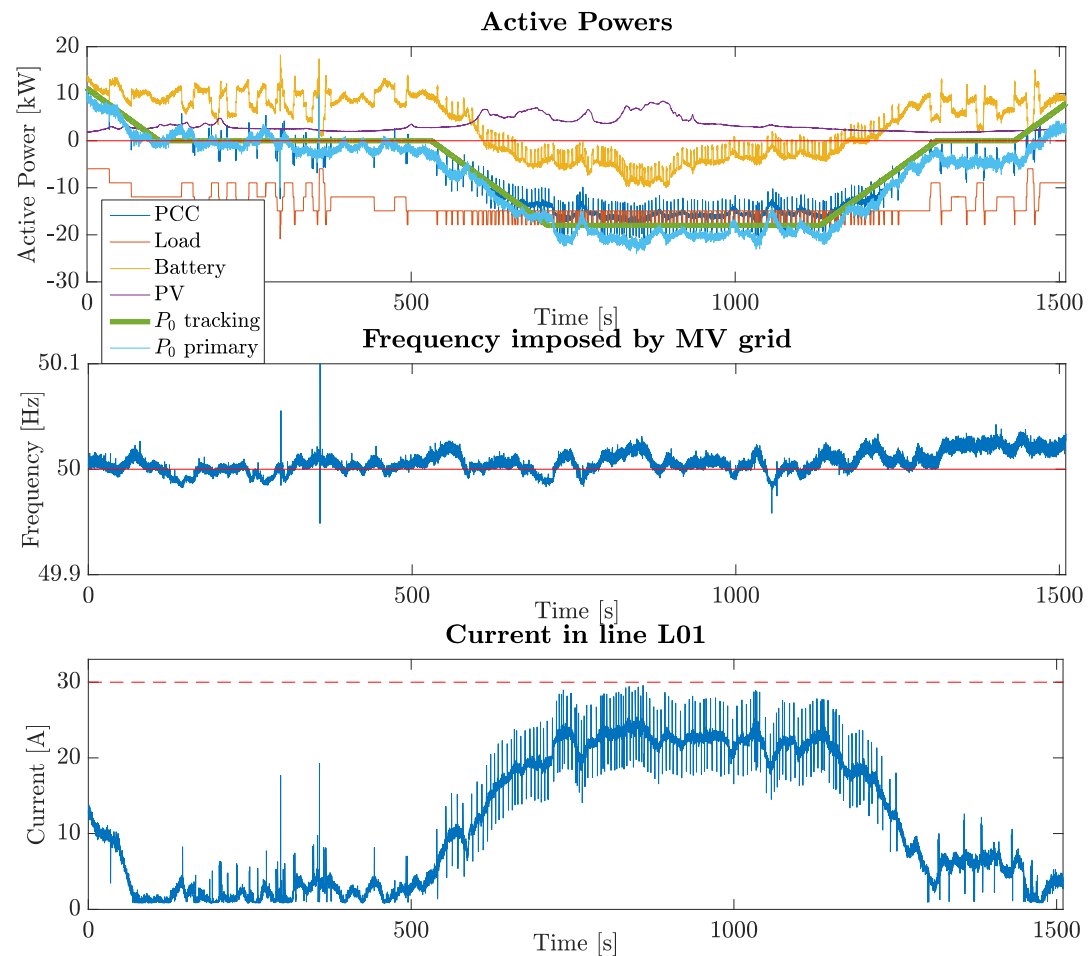
PMU and PDC  
data frame rate:  
50 fps



# Dispatch and Primary-Frequency Support

Superposition of dispatch and primary frequency control (i.e., primary droop control) with a max regulating energy of 200 kW/Hz

In parallel, keep the internal state of the local grid in a feasible operating condition.



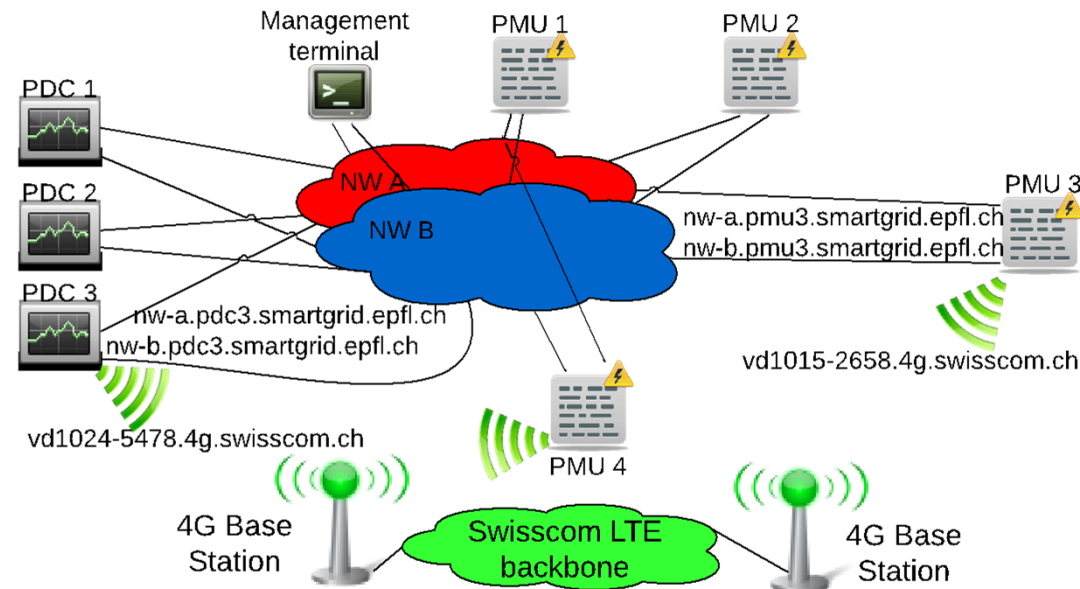
# COMMELEC Uses iPRP for UDP Packet Duplication

Controllers and sensors are connected to **2 independent networks**  
**iPRP** software duplicates packets at source and removes duplicates at destination

fully **transparent to application**  
– works with any application  
that streams UDP packets  
[Popovic et al 2016]

Open-source implementation:

<https://github.com/LCA2-EPFL/iprp>

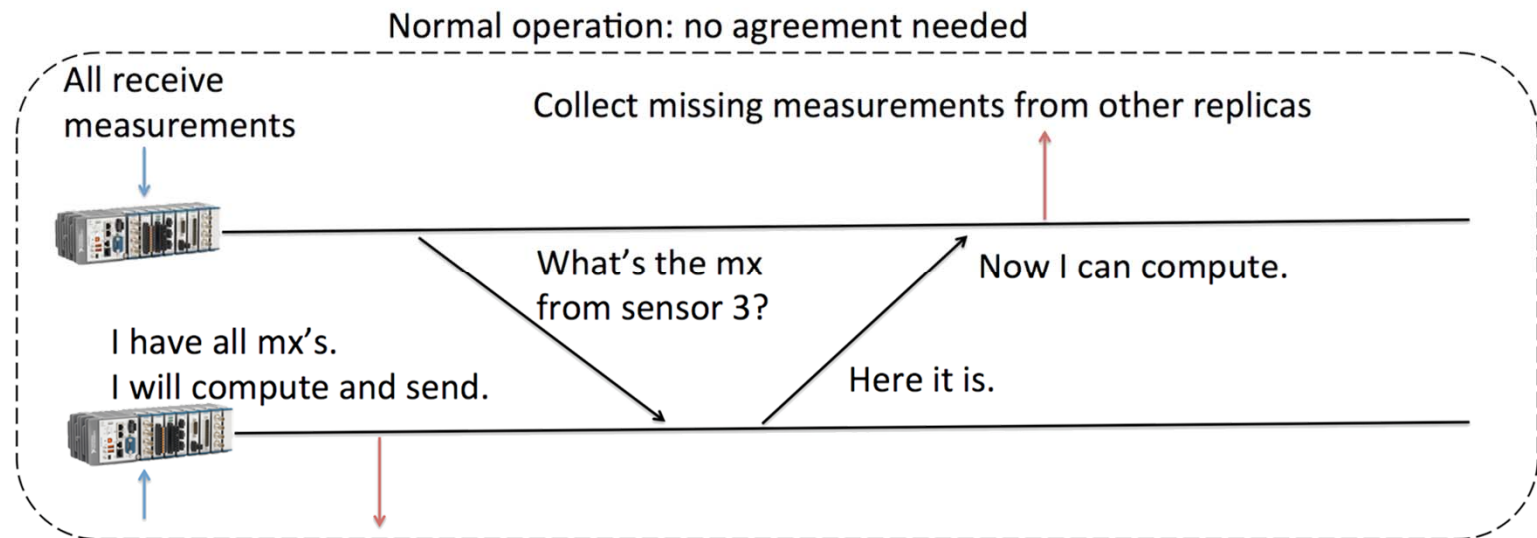


# COMMELEC Uses Active Replication with Real-Time Consensus

**Axo:** makes sure delayed messages are not used

**Quarts:** grid agents perform **agreement on input**

Added latency  $\leq$  one RTT – compare to consensus's unbounded delay  
[Mohiuddin et al 2017, Saab et al 2017]



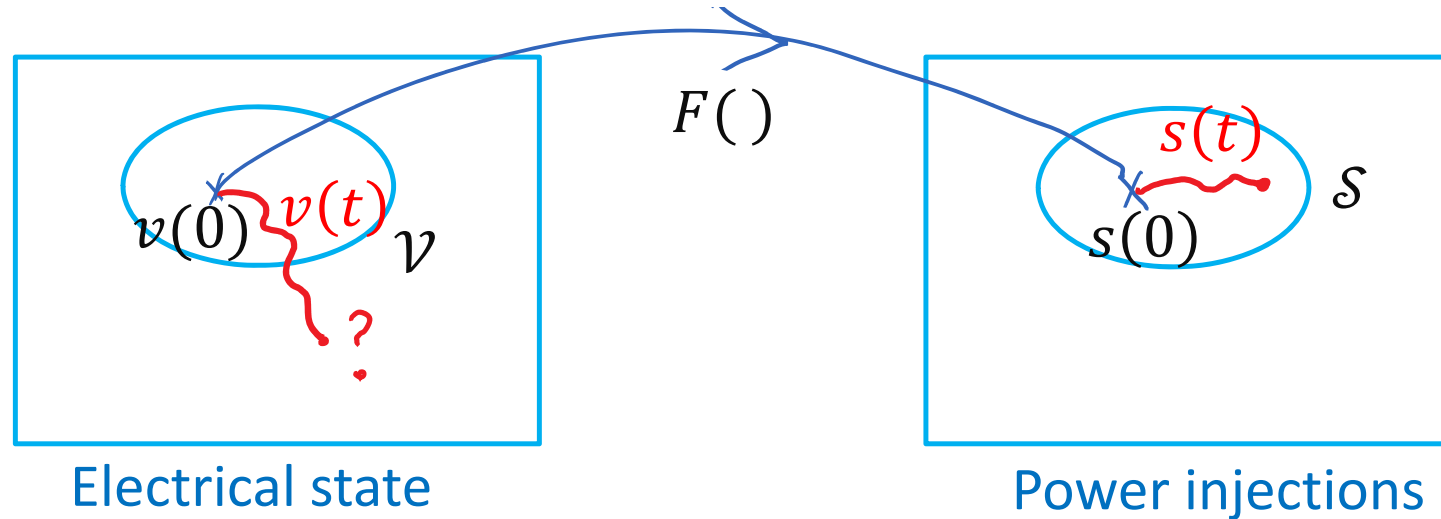
## 2. Controlling the Electrical State with Uncertain Power Setpoints [Wang et al 2017b]

**Admissibility test:** when issuing power setpoint  $x$ , grid agent tests whether the grid is safe during the next control interval for all power injections in the set  $S = BF(x)$ .

The abstract problem is:

- given an initial electrical state  $v$  of the grid
  - given that the power injections  $s$  remain in some uncertainty set  $\mathcal{S}$
- can we be sure that the resulting state of grid satisfies security constraints and is non-singular ?

# $\mathcal{V}$ -Control



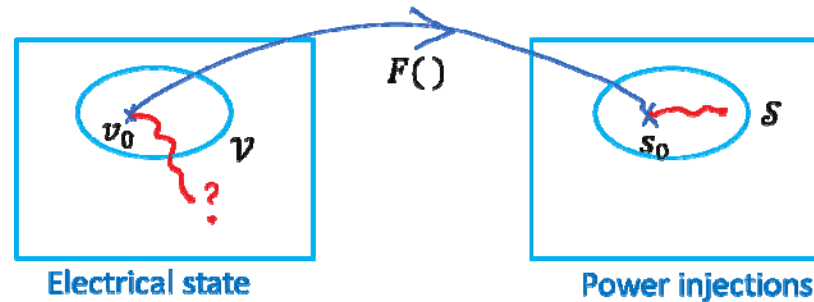
$\mathcal{S}$  is a **domain of  $\mathcal{V}$ -control**  $\Leftrightarrow$  whenever  $t \mapsto v(t)$  is continuous, knowing that  $v(0) \in \mathcal{V}$  and  $\forall t \geq 0, F(v(t)) \in \mathcal{S}$  ensures that  $\forall t \geq 0, v(t) \in \mathcal{V}$ .

3-phase grid with one slack bus and  $N$  PQ buses;  $v$  = electrical state = complex voltage at all non slack buses;  $s$  = power injection vector at all non slack buses

$s = F(v)$  is the power-flow equation

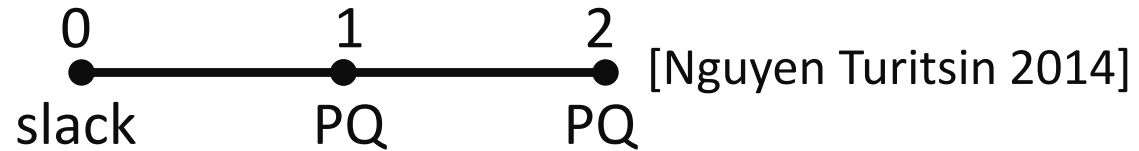
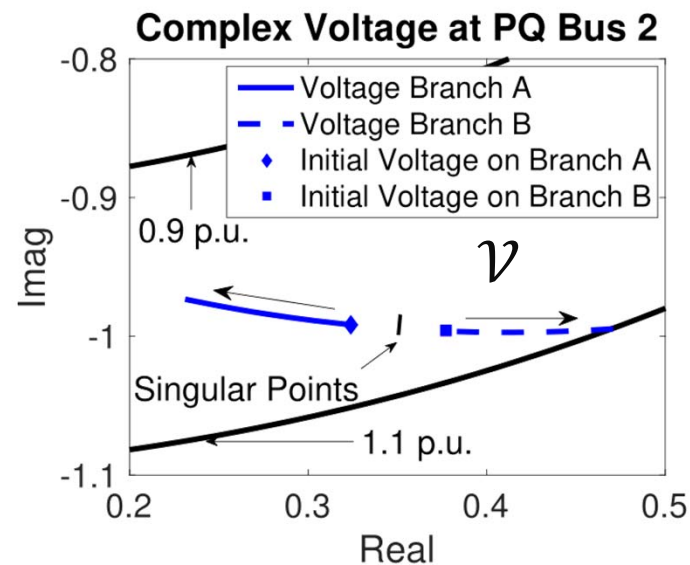
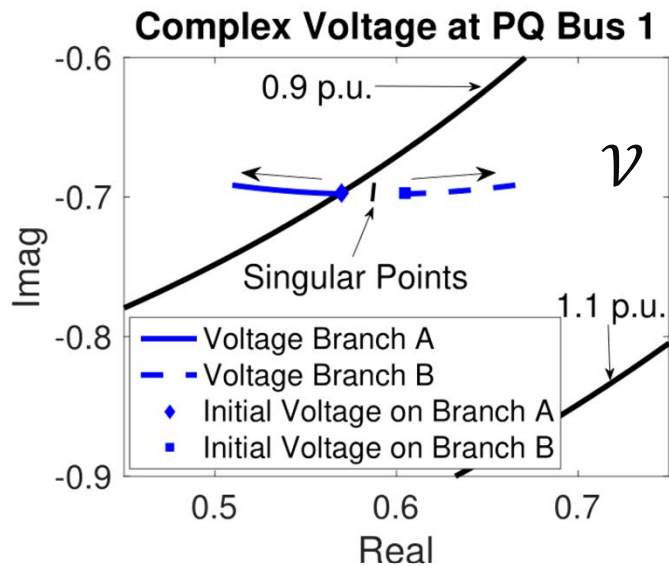
$\mathcal{V}$  is typically defined by voltage and ampacity constraints + non-singularity of  $\nabla F$

# Existence of Load Flow Solution Does not Imply V-control

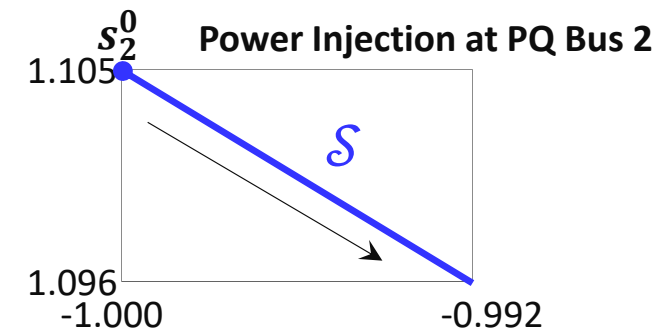
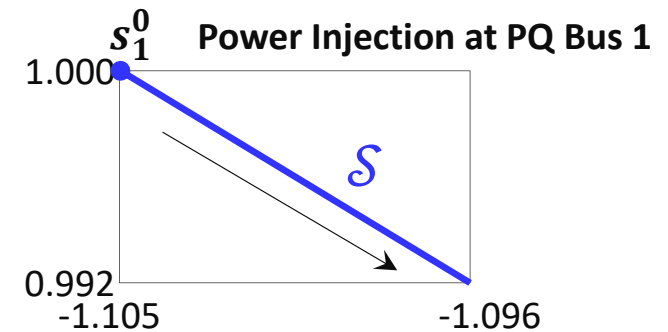


For  $\mathcal{S}$  to be a domain of  $\mathcal{V}$ -control it is necessary that every  $s \in \mathcal{S}$  has a load-flow solution in  $\mathcal{V}$ .

But this is not sufficient.



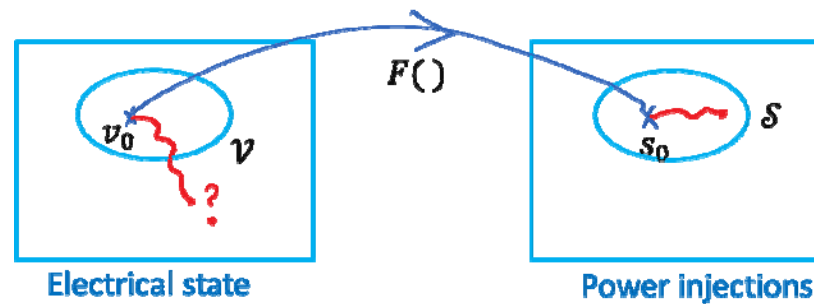
Every  $s \in \mathcal{S}$  has a load-flow solution in  $\mathcal{V}$ .  
But starting from  $s^0$  and  $v = \diamond$  we exit  $\mathcal{V}$ .



$\mathcal{V} = \{v: |v_1|, |v_2| \in [0.9; 1.1] \text{ and } \nabla F_v \text{ non singular}\}$   
 $\mathcal{S} = \{s = \kappa(s_1^0, s_2^0), \kappa \in [0.992; 1]\}$   
 $v = \diamond$  is in interior of  $\mathcal{V}$ , close to boundary (in  $s_1$ )

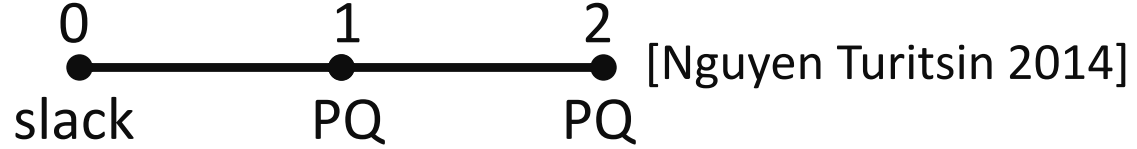
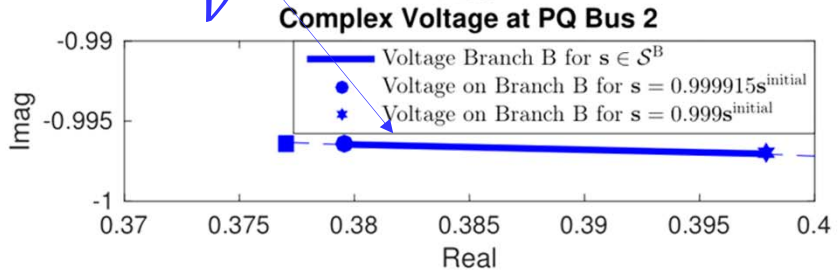
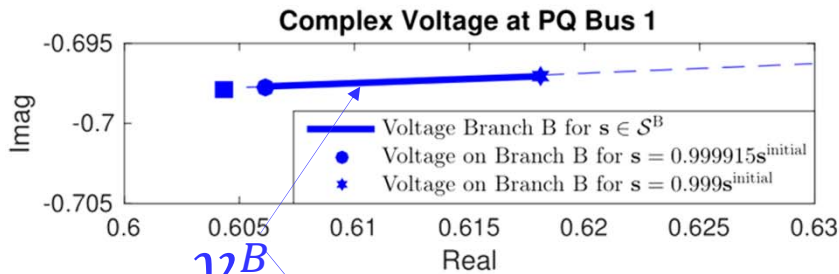
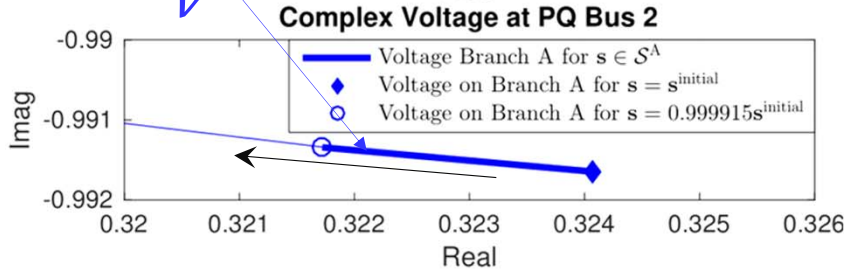
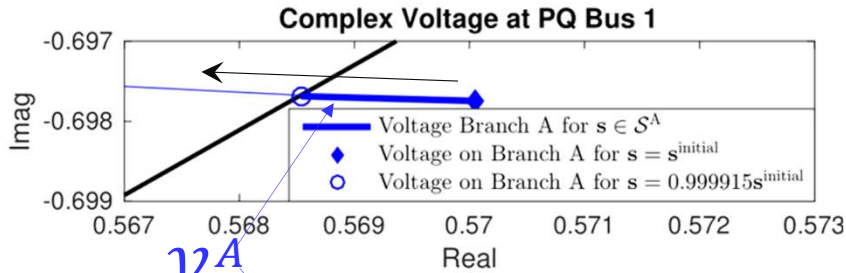


# *Unique* Load Flow Solution Does not Imply V-control



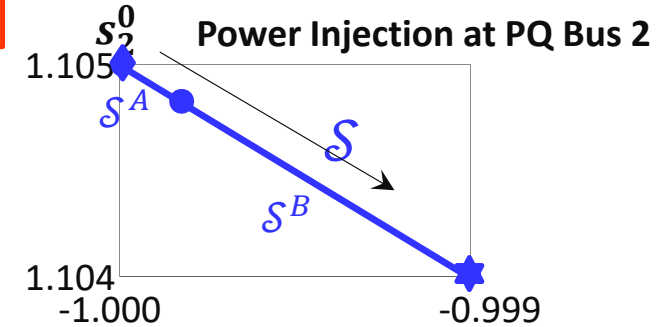
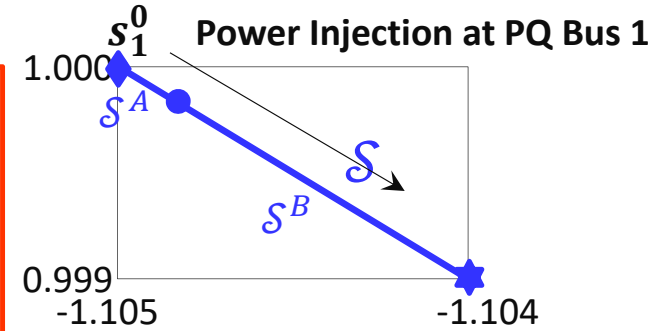
Assume that every  $s \in \mathcal{S}$  has a **unique** load-flow solution in  $\mathcal{V}$ .

This is **not sufficient** to guarantee that  $\mathcal{S}$  is a domain of  $\mathcal{V}$ -control.



Every  $s \in \mathcal{S}$  has a unique load-flow solution in  $\mathcal{V}$ .

But starting from  $s^0$  and  $v = \diamond$  we exit  $\mathcal{V}$ .



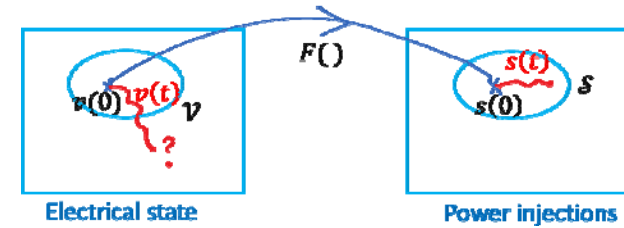
$$\mathcal{V} = \mathcal{V}^A \cup \mathcal{V}^B$$

$$\mathcal{S} = \{s = \kappa(s_1^0, s_2^0), \kappa \in [0.999; 1]\} = \mathcal{S}^A \cup \mathcal{S}^B$$

$$\mathcal{S}^A = \{s = \kappa(s_1^0, s_2^0), \kappa \in (0.999915; 1]\}$$

$$\mathcal{S}^B = \{s = \kappa(s_1^0, s_2^0), \kappa \in [0.999; 0.999915]\}$$

# Sufficient Condition for V-control

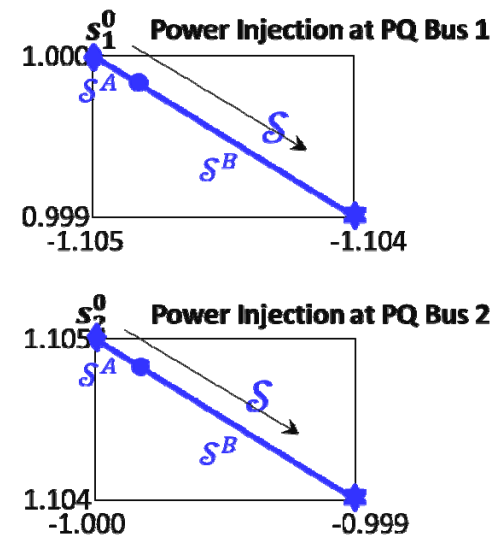


**Theorem 3** in [Wang et al 2017b]

If

1.  $\mathcal{V}$  is open in  $\mathbb{C}^{3N}$
  2.  $\mathcal{S}$  is open in  $\mathbb{C}^{3N}$
  3.  $\forall s \in \mathcal{S}$  there is a unique load-flow solution in  $\mathcal{V}$
- then  $\mathcal{S}$  is a domain of  $\mathcal{V}$ -control.

In the previous example, neither  $\mathcal{V}$  nor  $\mathcal{S}$  is open.



# V-control and Non-Singularity

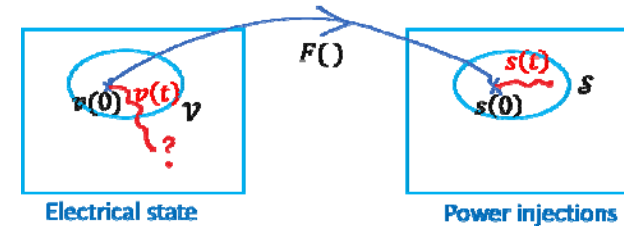
We call  $v$  **non-singular** if  $\nabla F_v$  is non-singular.

**Theorem 3** in [Wang et al 2017b]

If

1.  $\mathcal{V}$  is open in  $\mathbb{C}^{3N}$
  2.  $\mathcal{S}$  is open in  $\mathbb{C}^{3N}$
  3.  $\forall s \in \mathcal{S}$  there is a unique load-flow solution in  $\mathcal{V}$
- then  $\mathcal{S}$  is a domain of  $\mathcal{V}$ -control.

Furthermore, every  $v \in \mathcal{V}$  such that  $F(v) \in \mathcal{S}$  is **non-singular**.



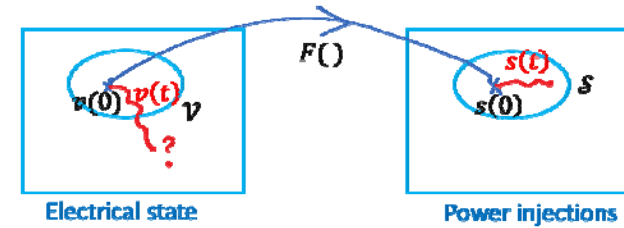
# Uniqueness and Non-Singularity

We call  $\mathcal{V}$  a **domain of uniqueness** iff

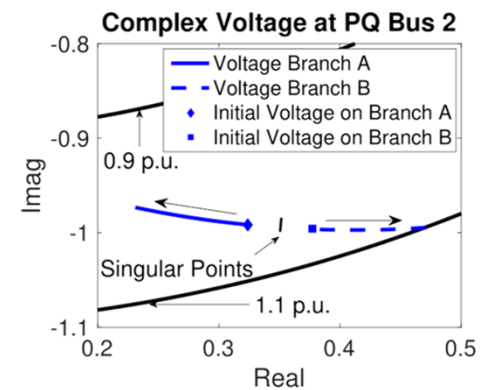
$$\forall v \in \mathcal{V}, \forall v' \in \mathcal{V}, v \neq v' \Rightarrow F(v) \neq F(v')$$

**Theorem 1** in [Wang et al 2017b]

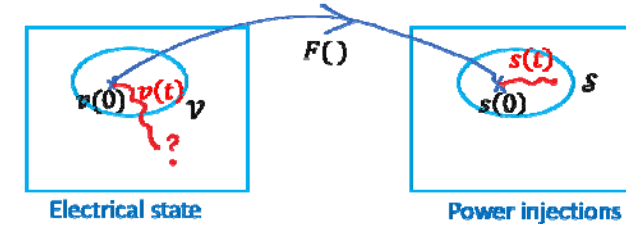
If  $\mathcal{V}$  is open in  $\mathbb{C}^{3N}$  and is a domain of uniqueness then every  $v \in \mathcal{V}$  is non-singular.



In this previous example,  $\mathcal{V}$  is not a domain of uniqueness



# Other Sufficient Condition for V-control

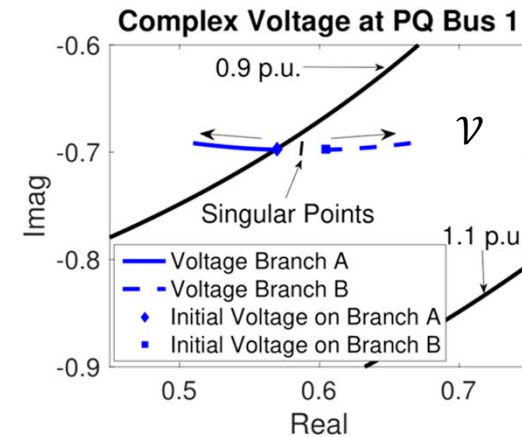


**Lemma 2** in [Wang et al 2017b]

If

1.  $\mathcal{V}$  is open in  $\mathbb{C}^{3N}$
  2.  $\mathcal{V}$  is non-singular
  3.  $\forall s \in \mathcal{S}$  there is a unique load-flow solution in  $\mathcal{V}$
- then  $\mathcal{S}$  is a domain of  $\mathcal{V}$ -control.

In the previous example  $\mathcal{V}$  is open but has singularities.



# Grid Agent's Admissibility Test, Re-Visited

**Problem (P):** Given a set of power injections  $\mathcal{S}^{uncertain}$ , find a set of electrical states  $\mathcal{V}$  such that

1.  $v(0) \in \mathcal{V}$
2.  $\mathcal{V}$  is open
3.  $\mathcal{V}$  is a domain of uniqueness
4.  $\mathcal{V}$  satisfies security constraints (voltages and line currents)
5.  $\mathcal{S}^{uncertain} \subseteq F(\mathcal{V})$

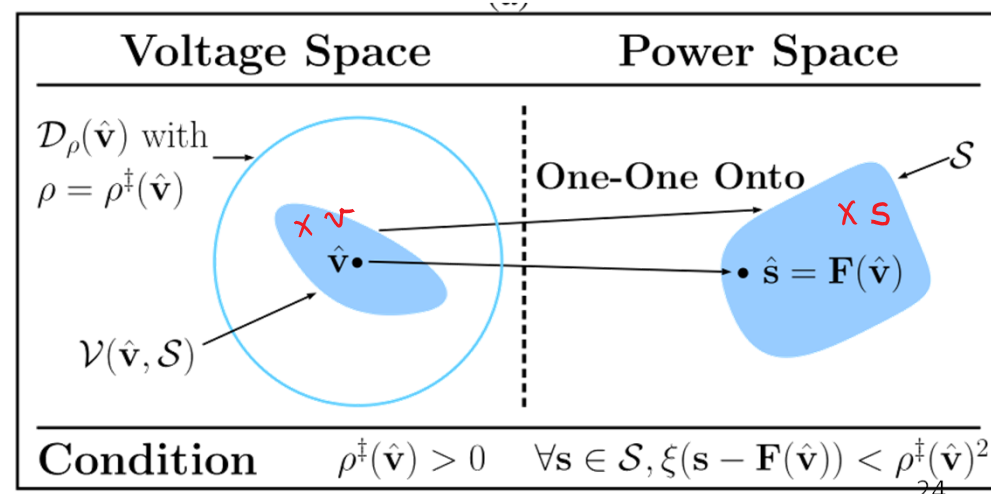
By Theorems 1 and 3 (applied to  $\mathcal{V}$  and  $\mathcal{S} = F(\mathcal{V})$ ), this will imply that  $\mathcal{V}$  is non singular and  $\mathcal{S}^{uncertain}$  is a domain of  $\mathcal{V}$ -control.

# Solving (P): Part A

Use sufficient conditions for uniqueness and existence of load flow.

**Theorem 1** in [Wang et al 2017a]

Given is a load-flow pair  $(\hat{v}, \hat{s})$ . If  $\xi(s - \hat{s}) < \rho^\ddagger(\hat{v})^2$  then  $s$  has a unique load flow solution in a disk around  $\hat{v}$  with radius  $\rho^\ddagger(\hat{v})$ . The norm  $\xi()$  and  $\rho^\ddagger$  are derived from the Y matrix.



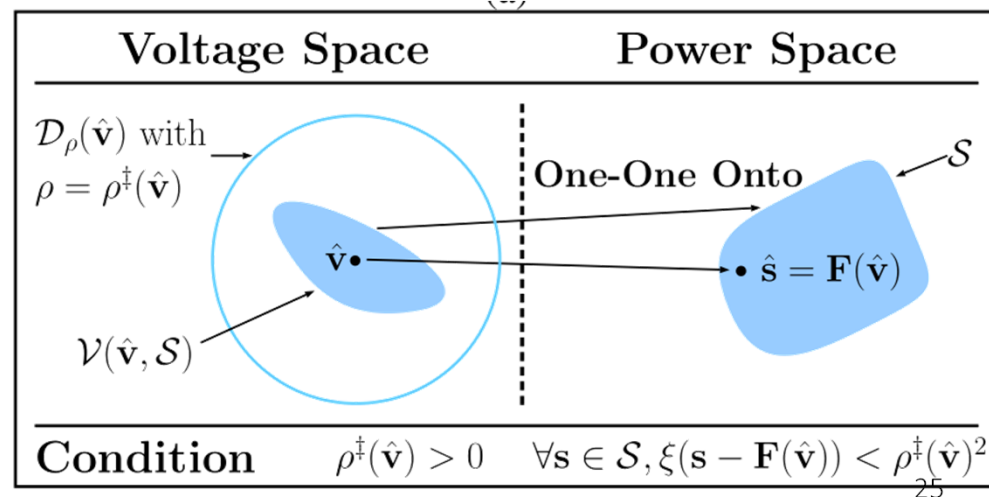


## Solving (P): Part A

Given is a set  $\mathcal{S}$  and a load-flow pair  $(\hat{v}, \hat{s})$  such that  $\hat{s} \in \mathcal{S}$ .

Assume **(C1)**  $\sup_{s \in \mathcal{S}} \xi(s - F(\hat{v})) < \rho^\ddagger(\hat{v})^2$

Then  $\mathcal{V}(\hat{v}, \mathcal{S}) = \{v \in \mathbb{C}^{3N}, F(v) \in \mathcal{S}\} \cap D_{\rho^\ddagger(\hat{v})}(\hat{v})$  is a domain of uniqueness.



## Solving (P): Part A

Given is a set  $\mathcal{S}$  and a load-flow pair  $(\hat{v}, \hat{s})$  such that  $\hat{s} \in \mathcal{S}$ .

Assume **(C1)**  $\sup_{s \in \mathcal{S}} \xi(s - F(\hat{v})) < \rho^\dagger(\hat{v})^2$

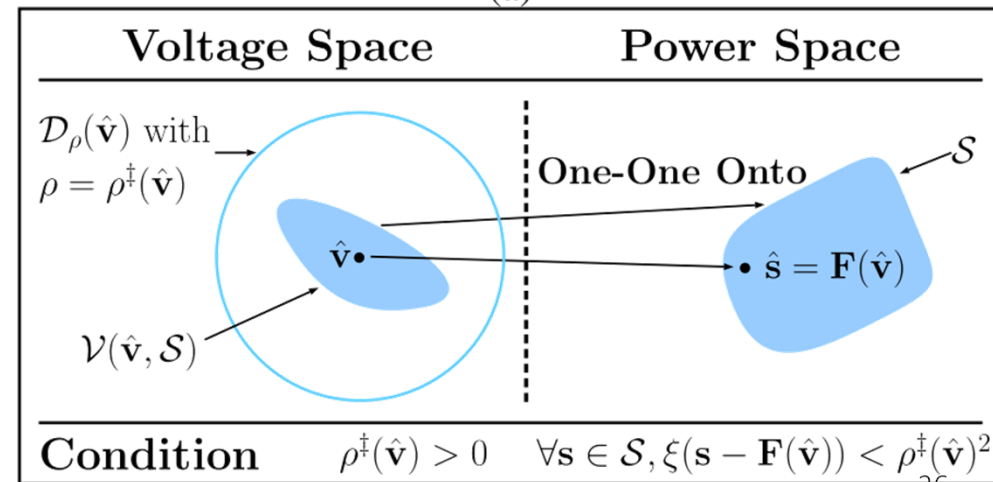
Assume in addition **(C2)** (Def.3 in [Wang et al 2017b])

$\sup_{s \in \mathcal{S}} \delta_j(s, \hat{v}) < \kappa_j$  for  $j = 1 \dots 6N$

Then  $\mathcal{V}(\hat{v}, \mathcal{S})$  is secured domain of uniqueness.

If  $\mathcal{S}^{uncertain} \subseteq \mathcal{S}$

then Problem (P) is solved !



## Notation [Wang et al 2017b]

$$\delta_j(\hat{\mathbf{v}}, \mathbf{s}) \triangleq \frac{\sum_{\ell=1}^N |\mathbf{\Gamma}_{j,\ell}| |\text{diag}(\mathbf{w}_\ell)^{-1}| \eta_\ell(\hat{\mathbf{v}}, \mathbf{s})}{u_{\min}(\hat{\mathbf{v}})(u_{\min}(\hat{\mathbf{v}}) - \rho^\dagger(\hat{\mathbf{v}}, \mathbf{s}))}; \quad (6)$$

zero-load nodal voltage  $\mathbf{w} \triangleq -\mathbf{Y}_{LL}^{-1} \mathbf{Y}_{L0} \mathbf{v}_0$

- $\mathbf{\Gamma}_{j,\ell}$ ,  $j, \ell \in \mathcal{N}^{PQ}$  is the  $3 \times 3$  submatrix formed by rows  $\{3j-2, 3j-1, 3j\}$  and columns  $\{3\ell-2, 3\ell-1, 3\ell\}$  of  $\mathbf{Y}_{LL}^{-1}$ ;

Notation	Definition
$\mathbf{W}$	$\text{diag}(\mathbf{w})$
$\xi(\mathbf{s})$	$\ \mathbf{W}^{-1} \mathbf{Y}_{LL}^{-1} \overline{\mathbf{W}}^{-1} \text{diag}(\overline{\mathbf{s}})\ _\infty$
$u_{\min}(\mathbf{v})$	$\min_{j \in \mathcal{N}^{PQ}, \gamma \in \{a,b,c\}}  v_j^\gamma / w_j^\gamma $
$\rho^\dagger(\mathbf{v})$	$\frac{1}{2} (u_{\min}(\mathbf{v}) - \xi(\mathbf{F}(\mathbf{v}))/u_{\min}(\mathbf{v}))$
$\rho^\dagger(\mathbf{v}, \mathbf{s}')$	$\rho^\dagger(\mathbf{v}) - \sqrt{\rho^\dagger(\mathbf{v})^2 - \xi(\mathbf{s}' - \mathbf{F}(\mathbf{v}))}$
$\eta_\ell(\mathbf{v}, \mathbf{s}')$	$u_{\min}(\mathbf{v})  \mathbf{s}'_\ell - \mathbf{F}_\ell(\mathbf{v})  + \rho^\dagger(\mathbf{v}, \mathbf{s}')  \mathbf{F}_\ell(\mathbf{v}) $

## Recap: Part A

Solve  $6N + 1$  optimization problems over the set  $S^{uncertain}$ .

The optimization problems are quasiconvex and maximum is always at a vertex.

If conditions (C1) and (C2) hold, then problem (P) is solved and admissibility test succeeds

(i.e. we can be certain that the grid will remain secured and non-singular as long as the power injections are in  $S^{uncertain}$ )

## Solving (P): Part B: Patching

Step A succeeds if we find an  $\mathcal{S}$  that covers  $\mathcal{S}^{uncertain}$ , which often works, but may fail when  $\mathcal{S}^{uncertain}$  is large.

Solution: **patching !**

**Theorem 6** in [Wang et al 2017b]: Assume we find a collection of pairs  $(\hat{v}_k, \mathcal{S}_k)$  such that  $\mathcal{S}^{uncertain} \subseteq \bigcup_k \mathcal{S}_k$  + condition (11) in [Wang et al 2017b]. Then the patching is consistent, i.e. the patchwork is a domain of uniqueness, secured, and non-singular and problem (P) is solved.

**Condition (11) in [Wang et al 2017b]**

**Definition 4.** Candidate pairs  $(\hat{\mathbf{v}}, \mathcal{S})$ ,  $(\hat{\mathbf{v}}', \mathcal{S}')$  are *consistent* if

$$\begin{aligned} & \|\mathbf{W}^{-1}(\hat{\mathbf{v}} - \hat{\mathbf{v}}')\|_{\infty} \\ & < \max\{\rho^{\ddagger}(\hat{\mathbf{v}}) - \sup_{\mathbf{s}' \in \mathcal{S}'} \rho^{\dagger}(\hat{\mathbf{v}}', \mathbf{s}'), \rho^{\ddagger}(\hat{\mathbf{v}}') - \sup_{\mathbf{s} \in \mathcal{S}} \rho^{\dagger}(\hat{\mathbf{v}}, \mathbf{s})\}. \end{aligned} \quad (11)$$



# Performance Evaluation

IEEE 37 bus feeder.  $\mathcal{S}^{uncertain} = [0, \kappa] \times$  benchmark values on all loaded phases. For  $0 \leq \kappa \leq 1.15$  algorithm declares  $\mathcal{S}^{uncertain}$  safe in one partition and <20 msec runtime on one i7; for  $\kappa > 1.15$  the algorithm needs multiple partitions but lowest voltage bound is close to limit.

IEEE 123 bus feeder.  $\mathcal{S}^{uncertain} = \left[1 - \frac{\kappa}{2}, 1 + \frac{\kappa}{2}\right] \times$  benchmark values on all loaded phases. For  $0 \leq \kappa \leq .31$  algorithm declares  $\mathcal{S}^{uncertain}$  safe in one partition and <30 msec runtime; for  $\kappa > .31$  the algorithm needs multiple partitions but highest branch current is close to limit.



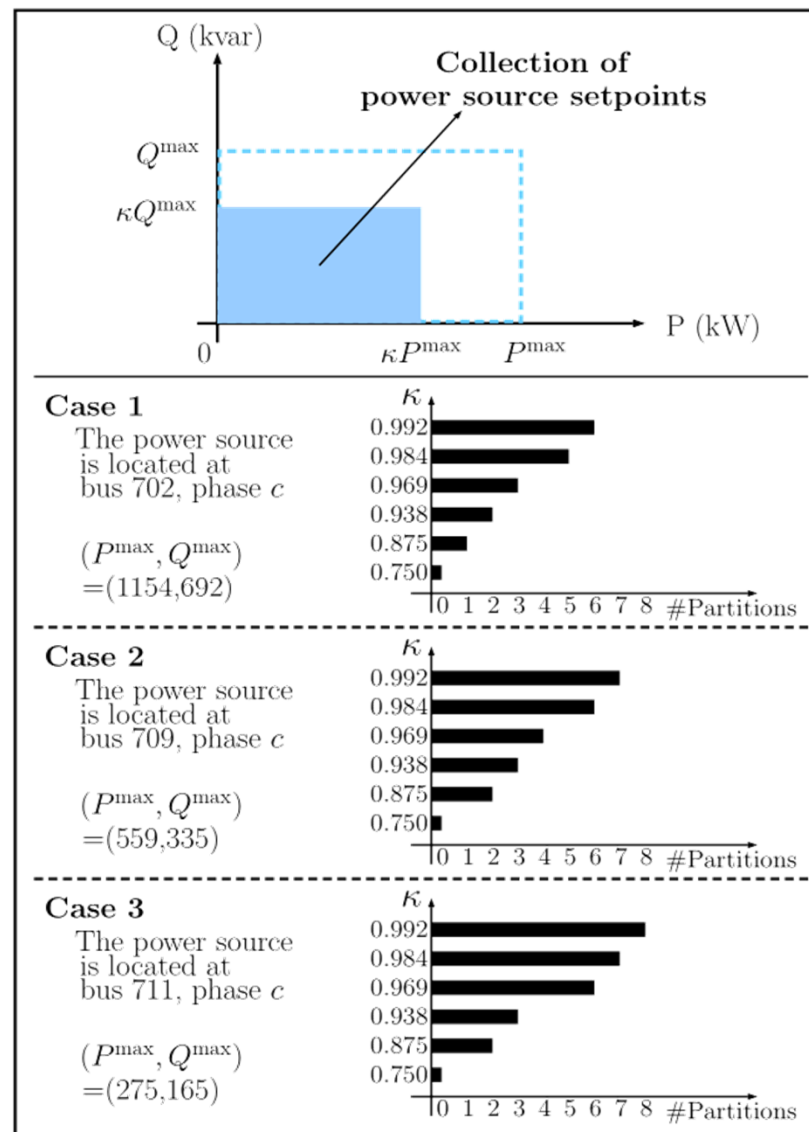
# Performance Evaluation

IEEE 37 bus feeder. One source added to one unloaded phase. Uncertainty set as shown. We limit the number of partitions to 8.

For  $\kappa \leq 0.750$  no partition.

For  $\kappa = 0.992$ , 8 partitions and run-time < 200 msec. Low voltage bound is close.

Incidentally, lowest voltage is not at  $(0,0)$  nor  $(P^{\max}, Q^{\max})$  (**non-monotonicity**)



# Conclusions

Controlling state of a grid by controlling power injections is needed in dynamic settings.

The theoretical problem is an inverse problem. It can be solved using the concept of **V-control**.

Uniqueness, existence, topological openness play an essential role.

A domain of uniqueness (of electrical states) is necessarily non-singular.

## References

- <http://smartgrid.epfl.ch>
- [Bernstein et al 2015, Reyes et al 2015a] Andrey Bernstein, Lorenzo Reyes-Chamorro, Jean-Yves Le Boudec , Mario Paolone, “A Composable Method for Real-Time Control of Active Distribution Networks with Explicit Power Setpoints, Part I and Part II”, in Electric Power Systems Research, vol. 125, num. August, p. 254-280, 2015.
- [Bernstein et al 2015b] Bernstein, A., Le Boudec, J.Y., Reyes-Chamorro, L. and Paolone, M., 2015, June. Real-time control of microgrids with explicit power setpoints: unintentional islanding. In PowerTech, 2015 IEEE Eindhoven (pp. 1-6). IEEE.
- [Mohiuddin et al 2017] Mohiuddin, M., Saab, W., Bliudze, S. and Le Boudec, J.Y., 2017. Axo: Detection and Recovery for Delay and Crash Faults in Real-Time Control Systems. IEEE Transactions on Industrial Informatics.
- [Nguyen Turitsyn 2014] Nguyen, H.D. and Turitsyn, K.S., 2014, July. Appearance of multiple stable load flow solutions under power flow reversal conditions. In PES General Meeting| Conference & Exposition, 2014 IEEE (pp. 1-5). IEEE.
- [Pignati et al 2015] M. Pignati et al , “Real-Time State Estimation of the EPFL-Campus Medium-Voltage Grid by Using PMUs”, Innovative Smart Grid Technologies (ISGT2015)
- [Popovic et al 2016] Popovic, M., Mohiuddin, M., Tomozei, D.C. and Le Boudec, J.Y., 2016. iPRP—The parallel redundancy protocol for IP networks: Protocol design and operation. IEEE Transactions on Industrial Informatics, 12(5), pp.1842-1854.
- [Reyes et al, 2018] Reyes-Chamorro, L., Bernstein, A., Bouman, N.J., Scolari, E., Kettner, A., Cathiard, B., Le Boudec, J.Y. and Paolone, M., 2018. Experimental Validation of an Explicit Power-Flow Primary Control in Microgrids. IEEE Transactions on Industrial Informatics.
- [Saab et al 2017] W. Saab, M. M. Maaz, S. Bliudze and J.-Y. Le Boudec. Quarts: Quick Agreement for Real-Time Control Systems. 22nd IEEE International Conference on Emerging Technologies And Factory Automation (ETFA), Limassol, Cyprus, 2017.015 IEEE World Conference on (pp. 1-4). IEEE.
- [Wang et al. 2016] Wang, C., Bernstein, A., Le Boudec, J.Y. and Paolone, M., 2016. Explicit conditions on existence and uniqueness of load-flow solutions in distribution networks. IEEE Transactions on Smart Grid.
- [Wang et al. 2017b] Wang, C., Bernstein, A., Le Boudec, J.Y. and Paolone, M., 2017. Existence and uniqueness of load-flow solutions in three-phase distribution networks. IEEE Transactions on Power Systems, 32(4), pp.3319-3320.
- [Wang et al. 2017b] Wang, C., Le Boudec, J.Y. and Paolone, M., 2017. Controlling the Electrical State via Uncertain Power Injections in Three-Phase Distribution Networks. IEEE Transactions on Smart Grid.